

777

THE
FIRST PRINCIPLES
OF
ARITHMETIC.

BY
A LATE TEACHER OF MATHEMATICS IN THE
ROYAL NAVY.

Et his principiis via ad majora sternitur.

NEWTON. QUAD. CURV.

L O N D O N:

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19

THE

FIRST PRINCIPLES

OF

ALGEBRA

BY

A LATE TEACHER OF MATHEMATICS IN THE

ROYAL ARTILLERY

By the Rev. J. H. COOPER, M.A.

OF THE UNIVERSITY OF OXFORD

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P R E F A C E.

AFTER the many elementary treatises already written, for the purpose of facilitating the study of numbers, the present compendium may to some, perhaps, appear superfluous, and unnecessary. A variety of Authors, improving on the labours of their predecessors, have indeed supplied the public with abundance of works upon this subject; but, though a new one is now added to the catalogue, the Editor of it does not mean, as is usually the case, to impeach the abilities of those who have gone before him in the same track; nor does he think it requisite to offer any apology for what he has done, especially as he solicits for no indulgence, and lays claim to no praise. From a very early period of life he had a strong propensity to the mathematics: by long study, and application, he acquired such a knowledge of them, as gave him reason to conclude, that he was sufficiently qualified to execute the task he undertook. He engaged in it on the request of a gentleman much conversant in the education of youth; and as it is now completed, he with deference submits the result of it to the opinion of those who may be competent to decide on its merits. Candidates for public favour ought to appear in the garb of modesty. The Author, therefore, will not attempt to prejudice his judges

judges by expatiating on the improvements he has made in this little treatise. It was professedly written for the use of schools, and, as such, he hopes it will be found useful. He must here, however, beg leave to observe, that to render it as much adapted as possible to the purpose intended, he has given as many examples, in each rule, as the limits of the work would admit; and that great care has been taken to preserve it from those errors which are too common in books of Arithmetic, and which both bewilder the scholar, and even oftentimes perplex the master.

ARITHMETIC.

PART I.

ARITHMETIC, as a science, is that mode of considering abstractedly the nature and properties of number, by which the truth of its various operations is demonstrated; and, as an art, it is the mode of performing these operations.

We have very little intelligence about the origin of Arithmetic, no history fixing either the author or æra.

It is very probable, however, that it was nearly coæval with civil society itself; and that they both derived their origin from necessity, the universal parent of invention: for man, stimulated by want, that most powerful incentive to all his desires, and led by an instinctive principle to discover the means of gratification, would soon think of applying combined efforts to obtain them. But the laws of inherent rectitude would immediately dictate, that the rewards should be in proportion to the toils: hence the necessity for some means to determine that proportion. Arithmetic, no doubt, was long practised as an art before it was considered as a science: for men would find it necessary to number, weigh, and measure, long before they became either mathematicians or philosophers; and, even in this scientific age, millions practise it as an art, who never dream of it as a science. Art generally paves the way to science, and science extends the limits of art: hence it is that Arithmetic is taught definitely and particularly, before it is taught indefinitely and universally.

The fundamental modes of operation by which we obtain all our arithmetical conclusions, are the following five: NOTATION, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION; of which in their order.

NOTATION.

NOTATION is the mode of expressing by symbols any number enunciated in words; and, vice versa, the mode of enunciating in words any number expressed by symbols.

Europeans at present express their numbers by the ten following symbols, 0 cypher, 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine; all which, except the
B cypher,

cypher, have two values, which may either be called symbolical and local, or simple and compound, the cypher serving only as an index to point out this last value. The simple value of each is expressed by its name, and is that alone which they possess when in the place of units; but this value is increased tenfold by every single removal from the place of units toward the left-hand, and decreased in the same proportion by every single removal from the place of units toward the right-hand: hence, by this increase or decrease, a new value is generated, which may be called local, as depending on its place; or compound, from its being compounded of the simple value and that power of ten, the index of which expresses its distance from the place of units: if therefore the several symbolical values be called numerators, and the powers of ten denominators, the local value of any symbol, however situated, will be expressed by the numerator and denominator conjointly.

The powers of ten are expressed in words and symbols, as in the following table; that being the more commodious symbolic form, where numerical indexes are used, in place of cyphers, to express the number of removals from the place of units.

TABLE OF THE POWERS OF TEN.

[illegible]

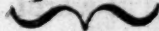
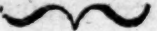
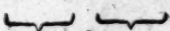
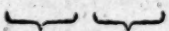
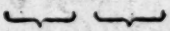
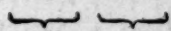
This table, which may be continued both ways, fine limite, is divided into periods and femiperiods, the period consisting of six places, and the femiperiod of three; the name of the first or central period is one or unit, that of the second, third, fourth, &c. on the left hand, million, bimillion, trimillion, &c. the compounded names being commonly contracted into billion.

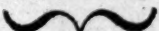
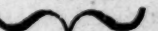
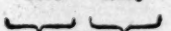
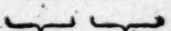
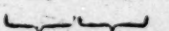
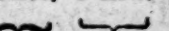
NOTATION.

3

lion, trillion, &c. the periods on the right hand have names similar to those equidistant from unity on the left hand, those on the left being named by the cardinal numbers, and these on the right by the ordinal.

Suppose now that any or all the places of cyphers were filled up with significant figures, divided and titled as underneath, a summary of the whole doctrine would be exhibited, and rules for solving the two following problems easily derived.

Septillions.	Sextillions.	Quintillions.	Quadrillions.
			
th. un.	th. un.	th. un.	th. un.
			
743,586.	678,974.	458,798.	258,653.

Trillions.	Billions.	Millions.	Units.
			
th. un.	th. un.	h. un.	c.x.t. c x.u.
			
586,497.	243,374.	983,278.	854, 279.

I. The nine digits are called significant, to distinguish them from the cypher, which is of itself insignificant.

II. Any number expressing a quantity of one name or denomination is called a simple number, and that representing a quantity of several denominations is called a compounded number.

III. Numbers which express no measures of either matter, motion, space, or time, or any of their properties or affections, are called abstract numbers.

IV. Numbers which represent quantities of the same kind are called homogeneous, but those which represent quantities of a different kind are called heterogeneous, and are neither capable of addition, subtraction, equality, nor any relation whatever.

PROBLEM I.

To enunciate in words any number expressed by symbols.

RULE.

To the simple value of each figure join the name of its place, reading from the left hand to the right.

NOTATION.

EXAMPLES.

Express in words the following numbers :

743567258942, 867428659563, 83247587948425,
 45672567901234586, 95867432458943, 25674325867834,
 7890356783583456, 973854967432734598,
 492586786743250879, 53792358734508678213,
 867423589767325407834328.

PROBLEM II.

To express by symbols any number enunciated in words.

RULE.

Write down the figures from the left hand to the right, in the same order they are enunciated.

EXAMPLES.

Express by symbols the following numbers.

Twenty-eight.

Six hundred and eighty-five.

Seven hundred and twenty-six thousand, eight hundred and forty-eight.

Six millions, four hundred and fifty-five thousand, seven hundred and thirty-six.

Eight billions, seven hundred and forty-eight millions, six hundred and fifty-four thousand, six hundred and eighty-six.

Four hundred and twenty-four thousand, six hundred and eighty-eight trillions, nine hundred and fifty thousand millions, seven hundred and sixteen thousand, five hundred and sixty-six.

Five hundred and eighty-six thousand quadrillions, seven hundred and forty-seven thousand, nine hundred and ninety-six trillions, seven hundred and ninety-eight thousand, four hundred and six billions, seven hundred and five thousand millions, four hundred and twenty units.

Ninety-eight thousand quintillions, seven hundred and four quadrillions, three millions, five hundred and seventy-seven thousand, nine hundred and ninety-four.

Sixty-four sextillions, seventy-eight quintillions, four quadrillion, ninety-five trillions, forty-eight millions, five hundred and six thousand, eight hundred and sixteen.

The Hebrews, Greeks, and Romans, each expressed their numbers by the letters of their own alphabet; and although we have now generally embraced the Arabian method, yet the Roman

Roman notation is still in use for some purposes, a specimen of which here follows.

A SYNOPSIS OF THE ROMAN NOTATION.

$$1 = \text{I}$$

$$2 = \text{II.} \left\{ \begin{array}{l} \text{As often as any character is repeated, so} \\ \text{many times its value is repeated.} \end{array} \right.$$

$$3 = \text{III.}$$

$$4 = \text{IV.} \left\{ \begin{array}{l} \text{A less character before a greater takes the} \\ \text{less value from the greater.} \end{array} \right.$$

$$5 = \text{V.}$$

$$6 = \text{VI.} \left\{ \begin{array}{l} \text{A less character after a greater adds the less} \\ \text{value to the greater.} \end{array} \right.$$

$$7 = \text{VII.}$$

$$8 = \text{VIII.}$$

$$9 = \text{IX.}$$

$$10 = \text{X.}$$

$$50 = \text{L.}$$

$$100 = \text{C.}$$

$$500 = \text{D. or IC.} \left\{ \begin{array}{l} \text{For every C affixed, this becomes ten} \\ \text{times as many.} \end{array} \right.$$

$$1000 = \text{M. or CIO.} \left\{ \begin{array}{l} \text{For every CO affixed, one on each} \\ \text{side, this becomes ten times as many.} \end{array} \right.$$

$$2000 = \text{MM.}$$

$$5000 = \overline{\text{V.}} \left\{ \begin{array}{l} \text{A line over any number, increases its value} \\ \text{a thousand fold.} \end{array} \right.$$

$$6000 = \overline{\text{VI.}}$$

$$10000 = \overline{\text{X.}} \text{ or } \text{CCICIO.}$$

$$50000 = \overline{\text{L.}} \text{ or } \text{ICCCIO.}$$

$$60000 = \overline{\text{LX.}}$$

$$100000 = \overline{\text{C.}} \text{ or } \text{CCCCICCCIO.}$$

$$1000000 = \overline{\text{M.}} \text{ or } \text{CCCCCICCCIO.}$$

$$2000000 = \text{MM.}$$

$$\&c. \quad \&c.$$

SIMPLE ADDITION.

SIMPLE ADDITION is the mode of collecting into one sum several homogeneous numbers of the same denomination; by performing which the following problem is solved.

PROBLEM III.

Any multitude of numbers, homogeneous, and of the same denomination, being given, to determine their sum.

RULE*.

1. Place all the numbers under each other in horizontal rows, so that places of the same power may be in the same vertical row; i. e. units under units, tens under tens, &c. and draw a line underneath.

2. Begin at the right hand, and add up all the figures in that first row; find how many tens are contained in their sum; set

* The method of arrangement and carrying is evident from the nature of notation; and the taking the sum of each vertical row, is founded upon a well-known and fundamental axiom—"the whole is equal to the sum of all its parts:" and as each row, and its respective remainder, are equidistant from the place of units, it follows that the number resulting from the several remainders thus arranged, is that required.

The first method of proof is founded upon the following theorem: The sum of the symbols retaining the same local value, however otherways arranged, is unvariable; thus, $a + b + c + d = d + c + b + a$.

The second method of proof is founded upon the above-mentioned fundamental axiom.

The third method of proof is founded upon the following theorem. Any number divided by nine, will leave the same remainder as the sum of its digits divided by nine; which may be thus demonstrated:

DEMONSTRATION.

Let N = any number whatever, founded upon the law of notation above mentioned, a, b, c , &c. the digits of which it is composed, and n = the number of places that a , the highest digit, is from that of units; then $N = a \times 10^n + b \times 10^{n-1} + c \times 10^{n-2}$, &c.

by the nature of notation $= a \times 10^n - 1 + a$

$+ b \times 10^{n-1} - 1 + b + c \times 10^{n-2} - 1 + c$, &c. =

$a \times 10^n - 1 + b \times 10^{n-1} - 1 + c \times 10^{n-2} - 1$ &c. $+ a + b$

$+ c$, &c. but $a \times 10^n - 1 + b \times 10^{n-1} - 1 + c \times 10^{n-2} - 1$ $+ ,$ &c. is divisible by nine without remainder: therefore N divided by nine will leave the same remainder as $a + b + c$, &c. divided by nine. Q. E. D.

SIMPLE ADDITION.

7

down what remains above the tens, or a cypher if nothing remains; and carry as many units to the second row as there were tens in the first.

3. Add up the second row, together with the number carried; find how many tens are contained in the sum; set down the remainder, and carry as before; and thus continue the process until the whole is finished, setting down the total amount of the last row: the resulting number will be that required.

METHODS OF PROOF.

Method I. Begin at the top, and add all the numbers downward, in the same manner that they were added upward; then, if the sums total be equal, the work is right.

Method II. Cut off the uppermost row by a line; add all the rest together, and set their sum under the number to be proved; if the sum added to the uppermost row, be equal to that found by the first addition, the work is right.

Method III. Add together the simple values of all the figures in each horizontal row severally; throw the nines out of their respective sums, and add their remainders together; if the excess of nines in this sum be equal to that in the sum total, the work is right.

EXAMPLES.

(1)	(2)	(3)	
74567524374	2432586432	5867496358	7
56743246732	5675867589	7876325867	5
49684325687	2867432648	4287250486	1
67432156758	6758943256	3258324567	0
32567532498	7459325487	6745894678	1
67432758056	9768423256	9032486749	7
35268974235	7325867473	7835642107	7
89674235867	8673249637	4583245678	7
473370754207	50961695778	49486666490	35
398803229833	50961695778	8	8
473370754207			

(4)	(5)	(6)
7567894325	256432586	895678907
8643258257	745863192	236589780
3249679524	574312875	325013627
6394208675	237890682	846732589
7543246758	423567458	789067458
6753624973	436798342	256786586
7894325878	658324867	923457897
2378016452	943768924	385649586

The mean distance of the sun from the several planets in our system being expressed in English miles as under, what is the sum of these distances in the same measure?

Georgium Sidus,	—	2252287132
Saturn,	—	1130911644
Jupiter,	—	616539904
Mars,	—	180623360
Earth,	—	118541428
Venus,	—	85744512
Mercury,	—	45881984

SIMPLE SUBTRACTION.

SIMPLE SUBTRACTION is the mode of finding the difference between any two homogeneous numbers of the same denomination; by performing which the following problem is solved.

PROBLEM IV.

Any two numbers, homogeneous, and of the same denomination, being given, to determine their difference.

RULE *.

1. Place the less number under the greater, so that places of the same power may be directly under each other; i. e. units under units, tens under tens, &c. and draw a line underneath.

2. Be-

* The method of arrangement here, as well as in Addition, is evident from the nature of notation; the other parts of the rule are

SIMPLE SUBTRACTION.

9

2. Begin at the right hand, and take each figure in the lower row from its corresponding figure in the upper row, and set the respective remainders underneath; but if any figure in the upper row be less than its corresponding figure in the lower row, increase, or suppose it to be increased, by ten, and then take the lower figure from it.

3. Set down the remainder, and carry one to the next figure in the lower row, with which proceed as before; and thus continue the process until the whole is finished: the number resulting from the several remainders thus arranged, will be that required.

METHODS OF PROOF.

Method I. Add the difference to the less number, and if the sum be equal to the greater the work is right.

Method II. Throw the nines out of the greater and less numbers, and also out of their difference; add the excess of the less number to that of the said difference: if this sum when less than nine, or its excess above nine, be equal to the excess of the greater number, the work is right.

are thus evident:—When all the figures in the upper row are greater than their corresponding figures in the lower row, it is very evident that the several remainders, arranged as per rule, will be the true difference; because the differences of the corresponding parts, similarly arranged, must be equal to the whole; and when any figure in the upper row is less than its corresponding figure in the lower row, the ten which is added is the value of an unit in the next higher place, by the nature of notation; and as the one which is added to the next figure in the lower row diminishes the value of its correspondent, the total difference remains unchanged: therefore the differences of the corresponding figures similarly arranged will be equal to the whole, as before.

The first method of proof is founded upon this self-evident truth, that the difference of any two numbers, added to the less, is equal to the greater:

The second method of proof is evident from the first of this, and the third method of proof in addition; because the greater number is equal to the less number and the remainder, and the excess of nines in the sum of two or more numbers is equal to the excess of nines in the digits composing those numbers: therefore the excess of nines in the greater will either be equal to the sum of the excesses in the other two, or equal to the difference between that sum and nine.

EXAM-

EXAMPLES.

From 74325867935 From 83586749632 From 986745867432
 Take 42312654622 Take 35867974286 Take 678576349875

Rem. 32013213313 Rem. 47718775346 Rem. 308169517557

Proof 74325867935 Proof 83586749632 Proof 986745867432

From 68210374583 From 96743081725 From 58402378678
 Take 43675843478 Take 64268198327 Take 32986735739

If the Christian Æra commenced Anno Mundi 4004, how long is it since the underwritten men of eminence lived?

A. M.

- 3097, Homer, the Greek poet.
- 3507, Pythagoras, founder of the Pythagorean philosophy.
- 3569, Pindar, the Greek lyric poet.
- 3672, Aristotle, the Greek philosopher.
- 3727, Euclid of Alexandria, the mathematician.
- 3734, Epicurus, founder of the Epicurean philosophy.
- 3740, Xeno, founder of the stoic philosophy.
- 3794, Archimedes, the Greek geometrician.
- 3849, Diogenes, of Babylon, the stoic philosopher.
- 3960, Vitruvius, the Roman architect.
- 3985, Virgil, the Roman epic poet.
- 3996, Horace, the Roman lyric poet.

A. C.

- 19, Ovid, the Roman elegiac poet.
- 25, Strabo, the Greek geographer.
- 1292, Roger Bacon, natural philosopher.
- 1582, George Buchanan, Scotch poet.
- 1550, John Napier, baron of Marcheston, was born.
- 1622, That famous discoverer of logarithms died.
- 1675, James Gregory of Aberdeen, mathematician.
- 1642, The illustrious Newton was born.
- 1727, That incomparable philosopher died.
- 1707, The great professor Euler was born.
- 1783, That profound mathematician died.

How

MULTIPLICATION.

11

How long is it since the underwritten memorable events?

A. M.

1770, The celestial observations begun at Babylon.

2289, Prometheus first struck fire from flints.

3284, The first eclipse of the moon on record.

3404, Maps, globes, &c. invented by Anaximander.

3957, The Alexandrian library burnt.

A. C.

49, London founded by the Romans.

1163, London Bridge first built of stone.

1340, Gunpowder invented by Swartz, a monk.

1494, Algebra first known in Europe.

1608, Satellites of Saturn discovered by Galileo.

1614, Logarithms discovered by Baron Napier.

1666, Fluxions discovered by Sir Isaac Newton.

1781, The Georgium Sidus discovered by Herschel.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

SIMPLE

SIMPLE MULTIPLICATION.

SIMPLE MULTIPLICATION is the mode of finding a number, such, that any two numbers of the same denomination being given, the number found shall contain any one of them so often as there are units in the other.

The number thus contained is called the *Multiplicand*.

The number shewing how often that is contained is called the *Multiplier*.

The number found is called the *Product*; by determining which the following problem is solved.

PROBLEM V.

Any two numbers of the same denomination being given, to determine a third that shall contain any one of them so often as there are units in the other.

RULE*.

1. Place the multiplier under the multiplicand, so that units may be under units, tens under tens, &c. and draw a line underneath.

2. Be-

* It is plain that the several products of the multiplicand by each of the figures in the multiplier are properly determined, supposing these figures to possess their simple values only: for the product of any two simple figures contains the one so often as there are units in the other, per definition; and the tens that are contained in any one of these products, will, by the nature of notation, become so many units in the next higher order; therefore each of the rows of products, on the above supposition, is properly determined.

Suppose now the several figures in the multiplier to change their local values, it is evident that their respective products will undergo a proportional change; therefore whatever place any figure of the multiplier possesses, the units place of the product, arising from its simple value, must possess a place of the same name, which is agreeable to the rule.

The methods of proof may be demonstrated in the following manner.

Method I.—Put m , n , and P = the multiplicand, multiplier, and product respectively; then, $m \times n = P$, $\therefore m = \frac{P}{n}$ and $n = \frac{P}{m}$; if the two last equations be multiplied by n and m respectively, they will become $m \times n = P$, and $n \times m = P$, therefore $m \times n = n \times m$. Q. E. D.

Method

2. Begin at the right-hand, and multiply every figure in the multiplicand by each of the figures in the multiplier, one after another, and place the products under one another in rows, thus :

3. Reckon how many tens there are in the product of the first figure in the multiplicand by each of the figures in the multiplier, and set down the remainder directly under the figure you are multiplying by ; carry as many units to the next product as there were tens in this last, and take the tens out of this increased product ; set down the remainder, and continue the process until the whole is finished.

4. Add all the products together, as they are now arranged, and the sum total will be that required.

METHODS OF PROOF.

Method I. Make the former multiplier the multiplicand, and the multiplicand the multiplier ; and if the product found from this operation be equal to that found from the former, the work is right.

Method II. Throw the nines out of the multiplicand and multiplier severally, and multiply the two excesses together ; if the excess of nines in this product be equal to that in the total product, the work is right.

EXAMPLES.

974325689356	856749674325
234	345000
3897302757424	4283748371625000
2922977068068	3426998697300
1948651378712	2570249022975
227992211309304	295578637642125000

Method II. Put M and N = the number of nines in the multiplicand and multiplier respectively, m and n their excesses ; then, $M + m$ = the multiplicand, and $N + n$ = the multiplier, and the product of those factors will be = $MN + Mn + Nm + mn$; but the three first terms are each a precise number of nines ; because one of the factors in each is 9 ; these therefore, being neglected, there remains mn to be divided by nine ; but mn is the product of the two former excesses : therefore the truth of the method is evident.

Q. E. D.
Multiply

Multiply 7432458315694 by 26. *Ans.* 193243916208044

Multiply 8543256741865 by 134. *Ans.* 1144796403409910

Multiply 5946759325686 by 496. *Ans.* 2949592625540256

Multiply 8567435674984 by 4283.

Ans. 36694326995956472

Multiply 9643258674897 by 8467.

Ans. 81649471200352899

Multiply 2438674567432 by 9658.

Ans. 23552718972258256

Multiply 7943267489325 by 24563.

Ans. 195110479340289957

Multiply 57345689458674 by 48972.

Ans. 2808333104170183128

Multiply 8745674234567 by 79435.

Ans. 694712632822829645

Multiply 64567494325867 by 823456.

Ans. 53168490607601136352

Multiply 49864789432586 by 943567.

Ans. 47050769770536874262

Multiply 67432586743285 by 4230564.

Ans. 285277873903018762740

Multiply 73258674325867 by 8674325.

Ans. 635469550171726264775

Multiply 30758006732586 by 60032580.

Ans. 1846482499814507651880

Multiply 67498708600325 by 74325670.

Ans. 5016886740853917842750

Multiply 86432586743258 by 234567890.

Ans. 20274309499608000785620

SIMPLE DIVISION.

SIMPLE DIVISION is the mode of finding how often one number is contained in another of the same denomination.

The number containing is called the Dividend.

The number contained is called the Divisor.

The number shewing how often the one is contained in the other is called the Quotient; by determining which the following problem is solved.

PROBLEM VI.

Any two numbers of the same denomination being given, to determine how often the one is contained in the other.

RULE.

RULE*.

1. Draw a curved line on the right and left of the dividend, and place the divisor on the left.
2. Find the number of times the divisor is contained in as many of the left-hand figures of the dividend as are just necessary, and place that number on the right.
3. Multiply the divisor by that number, and place the product under the above-mentioned figures of the dividend.
4. Subtract the said product from that part of the dividend under which it stands, and bring down the next figure of the dividend to the right of the remainder.

* According to the rule, the dividend is resolved into parts, out of which the divisor is taken one after another. It remains to be proved, that the several figures of the quotient taken as one number, according to the order in which they are placed, is the true quotient of the whole dividend divided by the divisor; which may be thus demonstrated.

Dem. The local values of the several parts into which the dividend is resolved, are as their distances from the place of units; but the several figures in the quotient, and the respective parts of the dividend from which they were taken, are equidistant from the place of units: therefore the quotient figures, arranged as per rule, is the true number required.

The first method of proof is evident from the nature of multiplication; because the dividend contains the divisor, either exactly or with some remainder, so often as there are units in the quotient; therefore it is plain that the product of the divisor and quotient, together with the remainder, if any, will be equal to the dividend.

The second method of proof is plain, granting the truth of the first; because the several rows of products, as they stand in the division, are the same with those in the multiplication, only placed in an inverted order, (i. e.) the first in the one is the last in the other, and vice versa, each respective figure possessing the same local value in the one as in the other.

The third method of proof may be demonstrated thus:

Dem. Let M and N = the nines in the divisor and quotient respectively, m and n = their excesses, and r = the remainder after division; then $M + m$ = the divisor, and $N + n$ = the quotient, the product of which, $+ r$ = the dividend = $MN + Mn + Nm + mn + r$; the three first terms of which being neglected, as a multiple of nine, there remains $mn + r$ to be divided by nine; but $mn + r$ is the product of the two former excesses together with the remainder: therefore the truth of the method is evident.

Q. E. D.

5. Divide the remainder, thus increased, as before, and if at any time it is found less than the divisor, put a cypher in the quotient, bring down the next figure of the dividend, and continue the process until the whole is finished, the quotient figures thus arranged will be that required.

METHODS OF PROOF.

Method I. Multiply the divisor by the quotient, or the quotient by the divisor; and if this product, together with the remainder, be equal to the dividend, the work is right.

Method II. Add together the remainder and all the products of the divisor by the several quotient figures, according to the order in which they stand in the work; and if this sum be equal to the dividend, the work is right.

Method III. Throw the nines out of the divisor and quotient separately, and multiply the two excesses together: add their product to the remainder, if any such there be, and throw the nines out of the sum; if the excess of nines in this sum be equal to that in the dividend, the work is right.

EXAMPLES.

Dividend.
Divis. 324)743256(2294 Quotient
648

952
648
—
3045
2916
—

1296
1296
—

Remain

Divisor 324
Quotient 2294

1296
2916
648
648
—

Proof 743256

Divide 3081440918904 by 12.	Ans. 256786743242
Divide 193243916208044 by 26.	Ans. 7432458315694
Divide 1144796403409910 by 134.	Ans. 8543256741865
Divide 2949592625540256 by 496.	Ans. 5946759325686
Divide 36694326995956472 by 4283.	Ans. 8567435674984
Divide 81649471200352899 by 8467.	Ans. 9643258674897
Divide 23552718972258256 by 9658.	Ans. 2438674567432

Divide

- Divide 195110479340289957 by 24563.
Ans. 7943267489325
- Divide 2808333104170183128 by 48972.
Ans. 57345689458674
- Divide 694712632822829645 by 79435.
Ans. 8745674234567
- Divide 53168490607601136352 by 823456.
Ans. 64567494325867
- Divide 47050769770536874262 by 943567.
Ans. 49864789432586
- Divide 285277873903018762740 by 4230564.
Ans. 67432586743285
- Divide 635469550171726264775 by 8674325.
Ans. 73258674325867
- Divide 1846482499814507651880 by 60032580.
Ans. 30758006732586
- Divide 5016886740853917842750 by 74325670.
Ans. 67498708600325
- Divide 20274309499608000785620 by 234567890.
Ans. 86432586743258

TABLES of COINS, WEIGHTS, and MEASURES.

ENGLISH MONEY.

Farthings.		
4 =	1 Penny.	} <i>2rs.</i> <i>D.</i> <i>S.</i> <i>£</i>
48 =	12 = 1 Shilling.	
960 =	240 = 20 = 1 Pound.	

TROY WEIGHT.

Grains.		
24 =	1 Penny-weight.	} <i>Gr.</i> <i>Dwt.</i> <i>Oz.</i> <i>lb.</i>
480 =	20 = 1 Ounce.	
5760 =	240 = 12 = 1 Pound.	

APOTHECARIES WEIGHT.

Grains.		
20 =	1 Scruple.	} <i>gr.</i> <i>ʒ</i> <i>ʒ</i> <i>ʒ</i> <i>lb.</i>
60 =	3 = 1 Dram.	
480 =	24 = 8 = 1 Ounce.	
5760 =	288 = 96 = 12 = 1 Pound.	

TABLES OF COINS, WEIGHTS, &c.

A VOIR DU POIS WEIGHT.

Drams.

16 =	1 Ounce.	} <i>dr.</i> <i>oz.</i> <i>lb.</i> <i>qr.</i> <i>cwt.</i> <i>T.</i>
256 =	16 = 1 Pound.	
7168 =	448 = 28 = 1 Quarter.	
28672 =	1792 = 112 = 4 = 1 Hundred weight.	
573440 =	35840 = 2240 = 80 = 20 = 1 Ton.	

W O O L W E I G H T.

Pounds.

7 =	1 Clove.	} <i>lb.</i> <i>Cl.</i> <i>St.</i> <i>T.</i> <i>W.</i> <i>Sa.</i> <i>La.</i>
14 =	2 = 1 Stone.	
28 =	4 = 2 = 1 Tod.	
182 =	26 = 13 = $6\frac{1}{2}$ = 1 Wey.	
364 =	52 = 26 = 13 = 2 = 1 Sack.	
4368 =	624 = 312 = 156 = 24 = 12 = 1 Last	

L O N G M E A S U R E.

Barley Corns.

3 =	1 Inch.	} <i>B.</i> <i>In.</i> <i>Ft.</i> <i>Yd.</i> <i>P.</i> <i>F.</i> <i>M.</i>
36 =	12 = 1 Foot.	
108 =	36 = 3 = 1 Yard.	
594 =	198 = $16\frac{1}{2}$ = $5\frac{1}{2}$ = 1 Pole.	
23760 =	7920 = 660 = 220 = 40 = 1 Furlong.	
190080 =	63360 = 5280 = 1760 = 320 = 8 = 1 Mile.	

S Q U A R E M E A S U R E.

Inches.

144 =	1 Foot.	} <i>In.</i> <i>Ft.</i> <i>Yd.</i> <i>P.</i> <i>R.</i> <i>Ac.</i>
1296 =	9 = 1 Yard.	
39204 =	$272\frac{1}{2}$ = $30\frac{1}{2}$ = 1 Pole.	
1568160 =	10890 = 1210 = 40 = 1 Rood.	
6272640 =	43560 = 4840 = 160 = 4 = 1 Acre.	

C L O T H M E A S U R E.

Inches.

$2\frac{1}{4}$ =	1 Nail.	} <i>In.</i> <i>Nl.</i> <i>Qr.</i> <i>Yd.</i> <i>F.E.</i> <i>E.E.</i> <i>Fr.E.</i> WINE
9 =	4 = 1 Quarter.	
36 =	16 = 4 = 1 Yard.	
47 =	12 = 3 = 1 Flemish Ell.	
45 =	20 = 5 = 1 English Ell.	
54 =	24 = 6 = 1 French Ell.	

COMPOUND ADDITION.

COMPOUND ADDITION is the mode of collecting into one sum several homogeneous numbers of different denominations; by performing which the following problem is solved.

PROBLEM VII.

Any multitude of numbers, homogeneous and of different denominations, being given, to determine their sum.

RULE*.

1. Place the numbers so that those of the same denomination may stand directly under each other, and the higher order on the left of the lower; then draw a line underneath.

2. Add up all the figures in the lowest denomination, and find how many units of the next higher order are contained in their sum.

3. Place down the remainder, and carry the units to the next higher order, which add up as before.

4. Proceed thus through all the various orders, or denominations, to the highest, the sum of which, together with the several remainders, will be that required.

The methods of proof are the same in this as the two first in Simple Addition.

EXAMPLES OF ENGLISH MONEY.

£	s.	d.	q.	£	s.	d.	q.	£	s.	d.	q.
743258	19	11	3	843274	14	06	2	964324	10	11	1
564325	16	10	2	674306	11	10	1	586396	11	10	2
874236	14	09	3	256732	15	11	2	458678	16	08	3
256758	13	11	1	749264	14	08	3	367459	17	10	2
678943	12	10	2	567458	16	10	0	685674	18	11	0
856757	17	08	3	456789	12	11	2	246739	12	10	3
497643	18	11	2	267432	17	09	0	587648	14	06	1
943014	15	10	0	583754	18	11	3	258754	19	11	0
732589	16	11	3	678923	10	10	2	890125	11	10	3
6147530	07	11	3	5077938	14	05	3	5045803	15	06	3

* The reasons for the arrangement, carrying, and methods of proof, in Compound Addition, are the same with those given for Simple Addition.

Simple and Compound Additions have only the following difference; in that the local increase is constant, in this it is variable.

N. B. In all numbers whatever, whether simple or compounded, the numbers which stand in the various orders or denominations, are called numerators, and the numbers which ascertain the values

COMPOUND ADDITION.

21

£	s.	d.	q.	£	s.	d.	q.	£	s.	d.	q.
87432567	18	11	2	8943258	14	03	2	94586745	14	09	3
74325674	16	10	2	7325674	15	08	1	73256874	15	11	2
68432567	17	11	3	2356749	16	10	2	67432567	16	10	3
25674567	16	10	2	7435679	14	11	2	74358674	15	10	2
48325674	12	08	3	6745748	12	07	2	67456745	16	11	2
68532568	14	00	1	4567325	10	08	2	73867430	12	10	1
97008674	15	10	2	7486749	18	11	1	20137358	10	11	2
74586749	16	11	3	2056756	16	10	3	74325674	12	08	1
73945675	18	11	2	7589432	17	06	3	10256749	17	11	2
23586743	12	10	1	4732567	15	10	2	68543258	18	10	1

EXAMPLES OF TROY WEIGHT.

lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.
75674325	11	18	22	476547	10	17	16
67432567	10	16	21	432586	11	00	21
74325674	08	12	20	743256	10	10	22
67432589	11	16	15	345678	11	12	13
56745678	10	12	08	1415679	08	03	17
67456743	08	17	18	8943235	11	17	12
45674567	11	16	21	5674567	09	18	18
73456748	09	15	16	3234567	10	13	22
94325674	10	16	17	2567439	06	16	11
83456748	10	12	12	2103587	11	18	20

lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.
8423476	11	18	14	74567432	08	12	21
6743258	10	19	21	67456743	05	16	19
7432586	11	16	12	45674325	11	17	18
4567432	09	11	13	91202132	11	18	19
7325867	10	16	17	41206743	10	16	12
8194325	11	13	20	67456874	06	17	22
7684325	11	14	12	86743258	10	13	21
9234567	00	16	08	74567432	11	12	20
7425678	10	19	22	94325874	02	10	00
3567894	06	16	12	67432586	11	12	20

of these numerators are called denominators; but in reduction, that name is given to the reciprocals of the proper denominators, merely to prevent circumlocution.

COMPOUND ADDITION.

lb.	oz.	dwt.	gr.
852347	06	13	10
674325	10	16	20
345674	11	13	14
514867	10	18	17
732456	11	19	21
643258	10	16	12
748325	08	10	10
325673	06	17	11
224867	11	12	21
367483	08	14	22

lb.	oz.	dwt.	gr.
47652347	11	19	22
32567437	01	12	21
74327674	10	13	23
67432456	11	18	21
73245674	10	12	20
72345647	09	19	19
45674323	11	12	01
70305860	01	02	10
89432586	10	13	14
53258674	11	16	12

EXAMPLES OF APOTHECARIES WEIGHT.

lb	3	3	3	gr.
54765476	11	7	2	19
74345678	10	3	1	16
74321458	10	1	0	17
32567867	11	0	1	02
67423246	03	6	2	11
32456748	10	7	1	03
74324567	11	3	2	11
10358674	10	2	1	13
14567423	11	1	2	16
74398674	00	3	1	18

lb	3	3	3	gr.
68532	09	6	1	18
25867	10	3	2	12
32567	06	6	1	09
23564	01	7	2	18
74324	11	4	1	12
54986	10	3	0	19
74324	10	2	0	16
87434	11	1	0	14
32568	03	6	1	15
78674	05	5	0	17

lb	3	3	3	gr.
765423	06	7	1	16
745674	01	6	2	18
645674	11	3	1	11
249467	10	1	0	12
745674	07	2	1	16
743256	11	5	1	17
632587	10	4	2	17
456732	08	3	1	19
743245	11	1	0	12
325674	10	0	1	14

lb	3	3	3	gr.
7432586	10	6	2	18
3245674	10	7	2	17
8674332	11	3	1	16
7432568	11	2	0	12
6743256	06	1	0	03
5325867	10	5	2	12
3580743	11	3	2	14
6745864	10	6	1	16
7432567	11	5	1	12
6432586	08	4	0	12

COMPOUND ADDITION.

23

lb	3	3	3	gr.
34765476	09	5	1	14
34567456	08	4	2	08
74008946	10	3	1	10
24356745	11	2	1	18
49567432	10	1	2	12
35843256	11	6	2	19
58674325	10	7	2	00
49367432	06	4	1	10
54374597	11	2	1	12
35456745	00	1	0	00

lb	3	3	3	gr.
523476	05	0	1	17
674325	10	6	1	18
732436	11	7	2	19
674347	06	3	1	10
235654	11	5	2	18
723567	05	4	1	12
832586	10	2	0	00
503007	00	1	0	08
743256	11	2	2	16
358605	10	7	2	18

EXAMPLES OF AVOIRDUPOIS WEIGHT.

Cwt.	qr.	lb.	oz.	dr.
5423476	3	27	12	14
7423437	2	26	13	12
7586743	2	18	15	12
8654329	3	18	14	10
2345678	0	22	12	00
5678567	1	24	13	13
8496758	2	22	12	11

Cwt.	qr.	lb.	oz.	dr.
5423476	2	24	13	12
7432587	1	21	14	11
6394358	2	22	11	10
7432567	3	18	10	12
6324586	2	16	00	01
7567325	1	20	11	12
3945678	2	22	12	13

Cwt.	qr.	lb.	oz.	dr.
7642346	1	10	12	13
3245867	2	20	11	12
5674358	3	22	12	13
1456789	2	19	14	11
7457632	1	19	15	12
6005806	2	08	14	03
7486798	3	21	15	04

Cwt.	qr.	lb.	oz.	dr.
54765476	3	24	12	13
67456743	2	18	10	12
34565742	1	10	15	12
73254356	2	14	10	11
67432542	1	15	15	14
76845235	2	17	12	10
82543245	3	10	14	11
35456742	1	14	12	09
89435643	2	10	10	00
10860703	1	12	12	13

COMPOUND ADDITION.

Cwt.	qr.	lb.	oz.	dr.
652347	3	27	15	15
745237	2	22	14	10
295674	1	21	12	11
743567	2	20	12	10
325868	1	19	10	10
464986	3	16	14	13
323543	1	17	14	00
725432	2	14	12	02
674358	1	16	14	03
945674	2	19	13	14

Cwt.	qr.	lb.	oz.	dr.
7654	2	22	08	09
6453	1	21	14	12
7325	2	22	15	15
5423	3	20	14	12
2546	3	24	12	10
3497	2	18	15	00
2345	1	16	00	10
3574	2	17	14	09
7548	3	14	10	11
5679	1	08	11	09

EXAMPLES OF WOOL WEIGHT.

L.	S.	W.	T.	S.	cl.	lb.
4324	11	1	6	1	1	0
4324	10	0	5	0	1	5
4235	11	1	4	1	0	4
8567	08	1	0	1	0	3
2456	10	0	6	0	1	4
5642	11	1	4	1	0	6
7849	10	1	3	1	1	3
8654	06	1	4	0	0	2
7543	08	0	2	1	1	6
4258	11	1	6	1	0	5

L.	S.	W.	T.	S.	cl.	lb.
4964	10	1	6	1	0	6
5675	10	0	4	1	1	4
2345	11	1	5	0	0	2
4564	10	1	6	1	1	3
5675	11	0	3	0	0	2
7432	09	1	4	1	1	5
2586	10	1	3	1	1	4
4967	11	0	1	0	0	4
5863	10	1	1	1	1	6
2345	11	1	6	1	0	3

L.	S.	W.	T.	S.	cl.	lb.
4325	11	1	6	1	0	6
8674	10	0	4	0	1	4
6745	11	1	3	1	1	3
2456	10	0	4	1	0	2
7357	11	1	2	1	1	6
3465	10	0	5	0	0	5
4356	11	1	6	1	1	6
7543	10	0	4	0	0	4
8958	11	1	6	1	1	0
5867	06	1	0	0	0	5
2643	10	1	1	1	1	6

L.	S.	W.	T.	S.	cl.	lb.
74325	10	1	6	1	1	4
45678	11	0	5	1	0	6
73425	10	1	4	0	1	5
24567	11	1	5	1	0	6
73258	10	0	6	0	1	3
67432	11	1	4	1	0	6
32567	10	0	6	1	0	5
86743	11	0	4	1	1	6
97435	10	0	2	1	0	5
64326	06	1	5	0	1	6

COMPOUND ADDITION.

25

L.	S.	W.	T.	S.	cl.	lb.	L.	S.	W.	T.	S.	cl.	lb.
4967	09	0	6	0	1	6	52347	10	1	5	1	0	5
3258	10	1	5	1	0	4	74235	11	1	6	0	1	6
4376	11	0	4	1	1	3	24589	10	1	0	1	0	3
2587	10	1	6	0	0	5	86743	06	0	6	1	0	6
7432	11	0	6	1	1	6	23567	10	1	4	0	1	5
3586	06	1	4	0	1	5	54324	11	0	5	1	0	6
6456	10	0	5	1	0	4	35467	05	1	2	0	1	4
5674	11	1	4	0	1	3	58674	10	1	4	1	0	6
2456	11	1	3	1	0	6	58763	11	0	3	1	1	4
5345	10	0	4	1	1	5	23576	11	1	6	1	1	5

EXAMPLES OF LONG MEASURE.

D.	M.	F.	P.	Yd.	F.	I.	B.	D.	M.	F.	P.	Yd.	F.	I.	B.
674325	59	7	39	5	2	11	2	25674	46	7	38	5	2	10	2
743258	48	6	36	4	1	10	2	67432	52	6	35	4	1	00	1
867432	34	4	32	3	0	11	2	56743	25	5	23	5	0	11	2
245678	49	6	37	5	1	10	1	74583	51	4	13	3	1	06	1
532456	54	7	32	1	2	08	0	43245	52	6	29	4	2	11	2
743245	52	4	24	4	1	06	2	74324	58	7	38	5	0	10	0
245867	35	2	25	5	2	11	1	68743	52	4	08	1	2	00	1
890674	29	5	38	4	1	10	2	34567	44	5	18	2	1	11	2
94345	56	0	09	0	1	04	0	49678	58	6	29	1	2	10	1
586953	58	4	32	4	2	11	2	54683	32	5	39	2	1	02	2

D.	M.	F.	P.	Yd.	F.	I.	B.	D.	M.	F.	P.	Yd.	F.	I.	B.
58674325	50	6	32	4	2	10	2	743258	58	4	39	5	2	09	1
74325670	03	4	31	2	1	08	2	456324	54	3	36	4	0	00	2
67432587	48	5	36	5	2	11	1	674358	49	4	31	4	1	02	1
23567458	51	4	36	4	1	01	2	793467	51	7	38	5	2	11	2
74325674	40	3	28	5	1	11	2	674358	48	6	24	3	1	10	1
67867456	41	6	35	4	0	10	2	786435	41	5	23	1	0	06	0
56745674	39	4	09	5	1	06	1	243207	35	0	06	5	1	10	2
64324567	45	3	34	4	2	11	2	456743	54	4	29	4	2	11	2
20103408	56	4	25	3	1	10	1	342567	51	3	30	2	1	09	1
13560543	51	5	21	1	0	11	2	735678	52	5	33	4	2	11	2

EXAMPLES OF CLOTH MEASURE.

<i>Yd.</i>	<i>Q.</i>	<i>N.</i>	<i>E. E.</i>	<i>Q.</i>	<i>N.</i>	<i>In.</i>	<i>Yd.</i>	<i>Q.</i>	<i>N.</i>	<i>In.</i>
68523476	3	3	423476	4	2	1	256743	3	3	2
74235674	2	2	674325	3	1	2	567435	2	2	1
25675323	1	0	567834	2	1	0	723458	3	3	2
74786235	2	1	724567	4	2	2	674325	2	1	2
45674324	3	3	673486	2	1	0	767246	1	3	2
24567456	1	0	274567	3	2	2	673458	3	1	1
23456753	0	2	567352	2	1	0	256754	0	2	0
42674325	2	1	763467	3	2	1	674349	2	3	1
73245256	3	2	876539	2	0	2	489678	1	0	2

<i>Fr. E.</i>	<i>Qr.</i>	<i>N.</i>	<i>In.</i>	<i>Fr. E.</i>	<i>Qr.</i>	<i>N.</i>	<i>In.</i>	<i>Yd.</i>	<i>Qr.</i>	<i>N.</i>	<i>In.</i>
74345674	5	3	2	7435674	2	2	1	256742	4	2	0
58674567	4	2	1	2567325	1	2	2	345678	2	1	2
83245678	2	1	2	5674586	0	0	1	732586	3	1	0
56753245	3	3	1	7003249	2	1	0	254324	1	2	2
24567456	5	2	2	3256758	1	2	2	624235	4	2	2
67894325	4	3	0	7423567	2	1	0	725786	2	1	1
74325678	3	2	2	2763456	1	0	1	243575	4	2	2
85674567	2	0	1	7235678	2	1	2	854906	2	1	0
94265310	0	2	2	6267432	1	2	2	200457	4	2	2
41024876	4	1	2	7352523	2	1	1	476775	3	1	2

EXAMPLES OF SQUARE MEASURE.

<i>A.</i>	<i>P.</i>	<i>Yd.</i>	<i>Ft.</i>	<i>A.</i>	<i>R.</i>	<i>P.</i>	<i>Yd.</i>	<i>Ft.</i>
74325673	39	30	7	73256	3	38	29	6
83254765	36	28	6	45674	2	33	30	1
24567432	23	30	8	32586	1	32	22	2
67305863	35	18	5	58674	3	36	25	4
32586749	24	00	4	24567	1	08	22	0
67432587	34	22	0	64658	2	19	16	1
94567456	22	17	6	32457	3	31	21	4
53474324	33	16	1	25675	1	21	22	7
94325765	20	21	3	86943	2	16	21	7
74567876	23	11	2	32568	3	22	19	8

COMPOUND ADDITION.

27

A.	R.	P.	Td.	Ft.
789567	2	33	26	7
567432	2	33	21	8
764325	1	38	11	0
773696	2	36	27	7
867535	3	22	21	6
786743	1	22	19	7
243567	2	19	16	0
356432	3	31	21	8
756874	1	20	19	6
325645	2	22	16	4

A.	R.	P.	Td.	Ft.
6743256	1	06	20	3
3242567	2	20	11	1
6732478	3	34	22	3
3428689	1	36	29	7
5456743	2	24	21	8
7587632	5	17	16	4
3958673	2	14	24	2
7325632	5	29	22	3
6732543	2	33	29	7
5943658	5	20	12	3

A.	R.	P.	Td.	Ft.
824325	3	34	24	6
743256	1	32	22	7
324567	2	28	21	3
763258	1	32	24	6
496734	2	16	18	4
943258	1	21	22	8
752367	0	09	15	4
328573	2	36	29	5
432645	3	24	16	7
567432	2	32	14	6

A.	R.	P.	Td.	Ft.
735867	1	33	25	3
423256	2	34	22	4
356743	3	21	20	7
867584	2	22	13	1
675843	1	29	27	2
567230	1	35	25	7
751245	2	29	24	6
245862	3	31	21	4
574567	1	30	20	0
473249	3	21	22	6

EXAMPLES IN WINE MEASURE.

Ft.	Hbs.	G.	Q.	P.
6	45674397	62	3	0
1	73256743	25	1	0
2	86732586	61	2	0
4	32543258	55	1	0
0	76432586	51	2	0
1	58673257	48	1	0
4	35768429	44	2	0
7	87325867	35	1	0
7	98674325	61	2	1
8	16758945	54	3	1

Tuns.	P.	H.	G.	Q.	Pt.
643258	1	1	60	2	1
734265	1	0	62	3	0
875873	1	1	58	2	1
678958	0	0	50	0	1
325867	1	1	48	3	0
437586	0	1	24	1	1
325867	1	0	22	2	0
789432	1	1	21	1	1
298783	1	1	61	0	1
732358	1	1	59	1	0

COMPOUND ADDITION.

<i>Tuns.</i>	<i>P.</i>	<i>H.</i>	<i>G.</i>	<i>Q.</i>	<i>Pt.</i>
768325	1	0	48	0	1
878675	1	1	50	1	0
632586	1	0	55	0	0
896735	0	1	51	2	1
674325	1	1	49	3	1
867298	1	0	56	1	0
732496	0	1	29	2	0
395867	1	0	60	1	0
496785	0	1	61	2	0
587432	1	0	52	1	1

<i>Tuns.</i>	<i>P.</i>	<i>H.</i>	<i>G.</i>	<i>Q.</i>	<i>Pt.</i>
767432	1	1	24	2	0
674324	0	0	40	1	1
395675	1	1	48	3	0
256758	1	1	51	1	0
586894	0	1	56	2	1
674325	1	0	61	3	0
895867	1	1	62	1	0
786783	1	0	51	2	0
258672	0	1	56	0	0
345742	1	1	60	2	1

<i>Tuns.</i>	<i>H.</i>	<i>G.</i>	<i>Q.</i>	<i>Pt.</i>
632586	3	29	1	1
743258	2	48	1	0
234567	3	49	2	1
768496	1	55	1	0
946858	3	56	1	1
497685	1	23	2	1
901050	2	61	2	0
358679	3	62	2	1
489735	2	44	1	0
894324	3	60	0	1

<i>Tuns.</i>	<i>P.</i>	<i>H.</i>	<i>G.</i>	<i>Q.</i>	<i>Pt.</i>
89586	0	0	55	2	0
73258	1	1	44	1	0
24567	1	0	61	2	0
49675	1	1	54	2	1
12348	0	0	62	3	0
74156	1	1	49	1	1
67587	1	0	54	2	0
24968	1	1	58	1	0
67587	0	1	42	3	0
49246	1	0	06	2	0

EXAMPLES OF ALE AND BEER MEASURE.

<i>Barr.</i>	<i>K.</i>	<i>F.</i>	<i>G.</i>	<i>Qt.</i>	<i>Pt.</i>
786743	1	1	8	3	1
247432	0	1	6	2	0
123245	1	0	5	1	1
745674	1	1	7	0	1
256758	1	0	4	1	0
586749	0	1	5	3	1
457498	1	1	3	2	1
678594	1	1	4	2	0
967849	0	0	6	3	1
586758	1	1	3	2	1

<i>Tuns.</i>	<i>B.</i>	<i>H.</i>	<i>G.</i>	<i>Qt.</i>
764324	1	1	51	3
568943	1	0	50	2
674895	0	1	48	1
324568	1	1	42	2
567874	0	1	41	3
742358	1	0	32	2
867432	1	1	44	3
274398	1	1	50	2
749867	1	1	31	1
100587	0	0	51	2

COMPOUND ADDITION.

29

Pt.	Butts.	H.	B.	K.	B.F.	Gal.
0	736743	1	1	1	1	8
1	258654	1	0	1	0	6
0	674325	1	1	0	1	4
0	235678	0	1	1	0	3
1	745867	1	0	0	1	2
0	867874	1	1	1	0	1
0	245643	0	1	0	1	7
0	856432	1	0	1	0	6
0	432458	1	1	0	1	8
1	967843	0	1	1	0	7

Butts.	H.	B.	K.	A.F.G.
743256	1	1	1	7
845342	0	1	0	6
956735	1	0	1	4
249674	1	1	1	5
673245	1	0	1	0
235674	1	1	0	1
456758	1	0	1	3
824379	1	1	0	6
973524	1	1	1	4
356743	0	1	0	5

Pt.	Butt.	H.	B.	K.	A.F.	G.	Qt.
0	742	0	1	1	1	7	3
0	683	1	0	1	0	5	2
0	732	0	1	0	1	4	3
1	256	1	1	1	1	6	2
0	743	1	1	1	1	4	2
1	135	0	1	0	0	3	0
0	758	1	1	1	1	2	1
0	824	0	0	1	0	4	3
0	256	1	1	1	0	7	2
0	325	1	0	0	1	6	2

Hbs.	B.	K.	B.	F.	G.
6892	1	1	1	1	8
5863	1	1	1	1	6
7684	1	0	0	0	5
4867	0	1	1	1	7
2456	1	1	0	0	4
5324	0	1	1	0	3
2456	1	0	0	1	7
5432	1	0	1	0	6
3245	1	1	0	1	8
2589	1	1	1	0	6

EXAMPLES IN DRY MEASURE.

Qt.	Last.	W.	Q.	Com.	Bu.	P.
3	76432	1	3	1	3	3
2	67458	1	2	0	2	1
1	68749	0	4	1	1	3
2	24586	1	2	1	0	2
3	73258	0	3	0	1	1
2	67324	1	2	1	2	3
3	78943	1	2	1	2	3
2	87694	0	1	0	3	0
1	96732	1	2	1	2	2
2	12456	1	3	1	3	3

Qrs.	Bu.	P.	Gal.	Qts.	Pts.
7689	7	3	1	3	1
4324	6	2	0	0	1
5675	3	3	1	2	0
8764	6	2	0	2	1
4573	2	1	1	3	1
6895	7	3	1	2	1
8943	6	2	1	1	0
2034	0	0	0	1	1
5867	5	3	1	3	1
7439	6	2	1	2	0

COMPOUND ADDITION.

Q.	Bu.	Pec.	Pt.	Qt.
7689	7	3	3	1
9432	6	2	2	0
6758	5	3	3	1
8949	4	2	2	0
7894	5	3	3	1
4267	4	2	2	0
3568	7	2	1	1
2456	3	1	2	0
6345	6	2	3	1
5891	7	3	2	0

Laß.	W.	Qr.	Com.	Bu.	P.
8392	1	4	1	3	3
5674	1	3	0	2	1
6745	0	1	1	2	2
7867	1	2	1	0	3
4568	1	4	1	1	2
2549	0	3	0	2	1
7364	1	0	1	1	0
2456	1	2	1	2	2
6732	0	1	0	3	1
7425	1	3	0	2	2
3549	1	3	1	1	3

Laß.	Qr.	Bu.	P.	G.	Qt.
794	9	7	3	0	0
946	8	6	2	1	3
567	6	3	0	0	2
456	7	3	3	1	3
245	8	2	2	1	2
734	6	3	1	1	0
689	2	0	0	0	3
456	7	4	1	1	2
325	6	3	1	0	2
567	8	6	1	1	3
258	7	4	1	1	2

Qr.	Bu.	P.	G.	Pt.
9432	7	3	1	7
5867	7	2	1	6
7432	6	3	0	5
4567	4	2	1	6
5675	6	2	0	7
6744	6	3	1	7
4325	5	2	1	2
2435	4	1	0	1
7324	3	1	1	4
5673	4	2	0	3
4965	6	3	1	4

EXAMPLES OF TIME.

Mon.	W.	D.	H.	M.	S.
6432	3	6	23	59	59
4286	1	3	22	51	41
2435	3	4	18	18	22
1748	2	5	22	45	41
5679	3	2	21	35	01
7486	1	5	22	30	31
8325	3	6	23	51	50
9678	2	5	21	49	41
4324	1	0	01	16	12
5145	2	4	24	25	22

Year.	Mon.	W.	D.	H.	Min.
7432	11	2	5	22	45
2428	12	3	6	21	50
4567	10	1	2	21	55
6735	11	2	4	20	50
4832	10	3	6	23	49
7435	11	2	5	22	50
4567	12	3	2	21	51
2674	12	2	5	14	07
3257	10	3	1	21	11
7325	11	2	4	22	18

COMPOUND ADDITION.

35

C.	Yr.	M.	W.	D.
732	99	12	3	5
456	96	11	2	4
572	85	10	1	6
784	84	10	2	5
698	72	09	1	4
567	64	10	2	3
245	54	11	3	4
567	88	06	1	0
705	18	02	3	4
568	94	12	2	6

Mon.	W.	D.	H.	Min.	S.
78676	2	4	22	40	50
87654	3	3	21	51	51
94324	2	4	20	52	49
67435	3	5	23	56	59
45674	2	6	21	14	00
56743	1	5	12	14	15
75867	2	3	21	22	21
67289	3	5	22	42	41
98904	2	4	12	09	19
87649	3	6	20	49	59

Year.	Mon.	W.	D.	H.	M.
2432	10	1	4	20	01
6743	11	2	6	22	40
5064	12	2	5	21	41
4675	10	1	6	22	49
7456	09	3	2	21	41
2589	12	2	1	09	19
6732	11	1	5	20	56
7458	10	2	4	22	49
5678	12	2	2	21	21
7948	10	3	5	19	24

C.	Yrs.	M.	W.	D.
4967	96	11	2	5
7439	94	10	3	2
5867	84	11	1	6
7425	72	10	2	4
5674	82	11	3	6
6789	94	12	2	5
8943	52	10	1	0
2586	51	12	3	3
4967	45	09	2	4
9089	22	10	1	5

COMPOUND SUBTRACTION.

COMPOUND SUBTRACTION is the mode of finding the difference between any two homogeneous numbers of different denominations; by performing which the following problem is solved.

PROBLEM VIII.

Any two numbers, homogeneous, and of different denominations, being given, to determine their difference.

RULE

RULE*.

1. Place the less number under the greater, so that those of the same denomination may stand directly under each other, and the higher order on the left of the lower; then draw a line underneath.

2. Begin at the right hand, and take each number in the lower row from its corresponding number in the upper row, and set the remainder underneath; but if any number in the upper row be less than its corresponding number in the lower row, increase, or suppose it to be increased, by as many as make one in the next higher order; subtract the lower number from the upper, and set the remainder underneath.

3. Carry one to the number in the lower row of the next higher order, with which proceed as before; continue the process until the whole is finished; and the several remainders taken together will be that required.

The method of proof is the same in this as the first in Simple Subtraction.

EXAMPLES OF ENGLISH MONEY.

	£.	s.	d.	q.
From	9786429	19	11	3
Take	4567234	12	06	1
Diff.	5219195	07	05	2
Proof	9786429	19	11	3

	£.	s.	d.	q.
From	8432567	14	08	2
Take	5861078	15	10	0
Diff.	2571488	18	10	2
Proof	8432567	14	08	2

	£.	s.	d.	q.
From	754235	11	03	0
Take	267856	14	08	3
Diff.	486378	16	06	1
Proof	754235	11	03	0

* The reasons for the arrangement, carrying, and method of proof, in Compound Subtraction, are the same with those given for Simple Subtraction, and are common to all the various denominations yet known.

COMPOUND SUBTRACTION

33

£.	s.	d.	q.
78567	16	10	2
43875	15	07	2

£.	s.	d.	q.
852316	10	11	0
486358	17	11	2

£.	s.	d.	q.
9675035	14	04	1
4864978	15	16	2

£.	s.	d.	q.
94325	17	11	2
47869	18	11	3

£.	s.	d.	q.
7432567	13	02	0
2496758	17	10	3

£.	s.	d.	q.
6749586	01	05	1
3258697	16	11	3

EXAMPLES IN TROY WEIGHT.

lb.	oz.	dwt.	gr.
9432586	10	18	22
3210452	09	14	16

lb.	oz.	dwt.	gr.
83569	11	19	23
45820	06	12	21

lb.	oz.	dwt.	gr.
73587	09	16	18
35605	02	13	11

lb.	oz.	dwt.	gr.
5867430	06	14	18
2948361	11	13	22

lb.	oz.	dwt.	gr.
754204	05	17	14
325820	08	18	20

lb.	oz.	dwt.	gr.
85674	10	06	04
35849	10	18	21

EXAMPLES IN APOTHECARIES WEIGHT.

℥	ʒ	ʒ	ʒ	gr.
956749	11	7	2	18
412354	10	5	1	14

℥	ʒ	ʒ	ʒ	gr.
49258	10	5	1	14
10325	03	20	0	12

COMPOUND SUBTRACTION.

£	3	3	9	gr.
67467	09	4	2	19
25678	02	1	0	14

£	3	3	9	gr.
742743	05	4	1	14
294268	10	5	2	16

£	3	3	9	gr.
56452	02	3	0	12
46749	06	0	2	16

£	3	3	9	gr.
49675	06	2	0	00
25867	10	2	1	18

EXAMPLES IN AVOIRDUPOIS WEIGHT.

T.	cw.	gr.	lb.	oz.	dr.
49325	16	2	25	12	14
32764	12	1	22	10	06

T.	cw.	gr.	lb.	oz.	dr.
9403	12	1	22	14	12
5672	10	0	14	10	10

T.	cw.	gr.	lb.	oz.	dr.
5736	18	2	27	12	13
3843	11	1	09	06	11

T.	cw.	gr.	lb.	oz.	dr.
56705	14	1	22	11	12
25816	14	2	23	13	14

T.	cw.	gr.	lb.	oz.	dr.
4298	03	0	16	09	10
2482	06	1	21	12	11

T.	cw.	gr.	lb.	oz.	dr.
964	13	1	14	02	09
573	19	2	18	10	12

EXAMPLES IN WOOL WEIGHT.

L.	S.	W.	T.	S.	cl.	lb.
7325	06	0	5	1	1	4
4967	11	0	4	0	1	2

L.	S.	W.	T.	S.	cl.	lb.
94325	11	1	6	1	1	6
67421	06	0	4	0	0	3

L.	S.	W.	T.	S.	cl.	lb.
75867	10	1	4	1	0	3
42358	07	0	2	1	0	2

L.	S.	W.	T.	S.	cl.	lb.
67432	06	1	4	0	0	4
49673	10	1	5	1	1	6

COMPOUND SUBTRACTION. 35

L.	S.	W.	T.	S.	cl.	lb.
96742	09	0	9	3	1	2
56894	11	1	4	0	1	5

L.	S.	W.	T.	S.	cl.	lb.
746742	04	0	6	0	0	1
257853	10	1	6	1	1	2

EXAMPLES IN LONG MEASURE.

Deg.	G.	M.	F.	P.	Yd.	Ft.	Deg.	G.	M.	F.	P.	Yd.	Ft.	In.
98674	59	7	39	5	2		6749	44	6	26	4	2	11	
45765	24	4	22	3	1		2586	21	3	18	2	0	06	

Leag.	M.	F.	P.	Yd.	F.	In.
4256	2	5	22	3	1	09
2467	1	2	20	0	0	04

Miles	F.	P.	Yd.	F.	In.	B.
6432	4	16	2	1	01	2
4570	5	24	4	2	10	1

Miles	F.	P.	Yd.	Ft.	In.	Bar.
8694	3	12	0	1	06	1
5489	6	34	2	2	10	2

Deg.	Miles	F.	P.	Yd.	Ft.
9840	56	3	26	3	1
4329	57	5	29	4	2

EXAMPLES IN CLOTH MEASURE.

Yd.	qr.	nl.	in.
9852347	3	3	2
4367429	1	2	0

E.	E.	qr.	nl.	in.
946432	4	2	2	
537586	3	0	1	

Fr.	E.	qr.	nl.	in.
965476	5	2	1	
586728	3	1	0	

Fl.	E.	qr.	nl.	in.
523047	1	1	0	
409868	2	3	2	

Fr.	E.	qr.	nl.	in.
956743	2	2	1	
245895	3	2	2	

Fr.	E.	qr.	nl.	in.
867302	2	1	0	
250826	4	2	1	

EXAMPLES IN SQUARE MEASURE.

Rds.	P.	Yd.	Ft.	In.
9463	39	30	8	120
5351	32	22	6	110

Ac.	Rd.	P.	Yd.	Ft.	In.
956	3	36	30	6	104
493	1	34	21	4	96

Ac.	R.	P.	Yd.	Ft.	In.
6425	1	21	22	5	122
4986	0	29	18	2	88

Ac.	R.	P.	Yd.	Ft.	In.
4961	1	16	22	4	99
1496	2	36	26	6	128

Ac.	R.	P.	Yd.	Ft.	In.
956	0	14	16	3	102
438	2	34	22	5	136

Ac.	R.	P.	Yd.	Ft.	In.
8901	0	21	12	4	48
2796	1	37	23	6	89

EXAMPLES IN WINE MEASURE.

Pun.	Tr.	G.	℥.	Pt.
27684	1	41	3	1
24765	0	37	1	6

T.	Btt.	Hbs.	G.	℥.	Pt.
958	1	1	36	2	1
425	1	1	24	1	0

Tuns	P.	H.	G.	℥.	Pt.
7586	1	1	61	3	0
4968	0	0	54	2	1

Pun.	T.	G.	℥.	Pt.
94325	0	21	2	0
43258	1	36	3	1

T.	Bu.	Hbs.	G.	℥.	Pt.
958	0	0	30	1	0
459	1	1	55	2	1

T.	B.	Hbs.	G.	℥.	Pt.
849	1	0	22	2	1
565	1	1	45	3	1

EXAMPLES IN ALE AND BEER MEASURE.

T.	H.	Ba.	K.	F.	G.	℥.	Pt.
978	3	1	1	1	7	3	1
496	2	1	0	1	4	1	1

T.	H.	Ba.	K.	A.	F.	G.	℥.	Pt.
642	2	0	1	1	6	2	0	
496	2	1	1	1	3	3	1	

Hbs.	Ba.	K.	B.	F.	G.	℥.	P.
567	1	1	1	8	2	1	
249	0	0	1	6	1	0	

Hbd.	Ba.	K.	B.	F.	G.	℥.	P.
95243	0	1	0	4	1		
26738	0	1	1	6	2		

COMPOUND SUBTRACTION.

37

Pun. K. A. F. G. Q.

9476 0 1 4 0
3587 1 1 5 2

T. Hbs. G. Q. P.

93246 2 24 2 0
57587 3 58 3 1

EXAMPLES IN DRY MEASURE.

Chal. Cm. Pc. G. Pot. Q. P.

9685 3 13 1 1 1 1
4396 2 11 0 1 0 1

Qr. C. Pc. G. Pot. Q. P.

742 0 11 1 1 1 0
467 1 10 1 0 1 1

Chal. Qr. B. Pc. Pot. P.

843 3 7 3 3 3
694 1 5 2 1 2

Chal. Qr. B. Pc. Pot. P.

9567 1 4 1 2 1
2986 2 6 2 3 3

Chal. Cm. Pc. Pot. P.

486 4 12 1 1
199 6 13 2 3

Last. W. Qr. B. G. P.

942 1 2 5 4 5
429 1 4 7 5 7

EXAMPLES IN TIME.

Cent. Yr. Mon. W. D. H.

978 99 12 3 6 23
459 81 06 1 4 12

Yr. Mon. W. D. H. M.

496 11 2 4 21 41
289 07 1 2 20 22

Mon. W. D. H. M. Sec.

943 1 5 12 36 33
458 2 3 10 29 45

Mon. W. D. H. M. Sec.

674 1 2 11 45 51
286 2 5 14 46 55

Yr. M. W. D. H. M. Sec.

784 09 1 3 19 22 31
468 10 2 4 21 37 52

Cent. Yr. M. W. D. H. M.

931 41 03 1 5 12 13
145 69 10 2 6 21 16

COMPOUND MULTIPLICATION.

COMPOUND MULTIPLICATION is the mode of finding a number, such, that any two numbers, a simple and compounded, being given, the number found shall contain the compounded so often as there are units in the simple.

The compounded number is called the Multiplicand.

The simple number is called the Multiplier.

The number found is called the Product; by determining which the following problem is solved.

PROBLEM IX.

Any two numbers, a simple and compounded, being given, to determine a third that shall contain the compounded so often as there are units in the simple.

RULE*.

1. Multiply the lowest denomination of the Multiplicand by the first figure in the multiplier; find how many units of the next higher order are contained in the product, and set the remainder underneath; carry the units thus found to the product of the next higher order, with which proceed as before, and so on to the highest.

2. Multiply in the same manner by every figure in the multiplier, one after another, until the whole is finished.

* Simple and compound multiplication differ only in the law of local increase; in that it is constant, in this it is variable. From this consideration, joined to what has already been said on the foregoing rules, the first four parts of this rule are evident.

The last part is founded upon the following theorem: Any number is equal to the continual product of all its component factors; which may be thus demonstrated:

DEM. Let N = any number whatever, which contains m so often as there are units in n ; also let m contain a so often as there are units in b , and n contain c so often as there are units in d ; I say that $N = a \times b \times c \times d$: because $N = m \times n$, and $m = a \times b$, and $n = c \times d$: therefore substituting for m , and n , their respective values, there results $N = a \times b \times c \times d$. Q. E. D.

The above is true, whether the factors are whole, fracted, or surd; but those mentioned in the above rule, are understood to be whole numbers, the units in each not exceeding twelve.

3. Multiply each of the above products by that power of ten the index of which expresses the distance of its respective generating figure in the multiplier from the place of units.

4. Arrange the last products as in compound addition, and the sum of the whole will be that required.

5. But if the multiplier is the product of several single factors, the continual product of the multiplicand by these several factors will be that required.

The method of proof is the same in this as the first in simple multiplication.

EXAMPLES IN MONEY.

1. 6 yards of cloth, at *ol.* 18*s.* 6*d.* 2*q.* per yard.
Ans. 5*l.* 11*s.* 3*d.* 0*q.*
2. 7 hundred of cheese, at 3*l.* 10*s.* 6*d.* per cwt.
Ans. 24*l.* 13*s.* 6*d.* 0*q.*
3. 8 firkins of butter, at 1*l.* 18*s.* 6*d.* 3*q.* per firkin.
Ans. 15*l.* 8*s.* 6*d.* 0*q.*
4. 9 quarters of wheat, at 1*l.* 4*s.* 6*d.* 2*q.* per quarter.
Ans. 11*l.* 0*s.* 10*d.* 2*q.*
5. 10 anchors of brandy, at 2*l.* 12*s.* 11*d.* 3*q.* per anchor.
Ans. 26*l.* 9*s.* 9*d.* 2*q.*
6. 11 hogsheads of porter, at 8*l.* 8*s.* 6*d.* 1*q.* per hoghead.
Ans. 92*l.* 13*s.* 8*d.* 3*q.*
7. 12 tuns of wine, at 100*l.* 18*s.* 2*d.* 2*q.* per tun.
Ans. 1210*l.* 18*s.* 6*d.* 0*q.*
8. 14 yards of broad cloth, at 1*l.* 12*s.* 6*d.* 3*q.* per yard.
Ans. 22*l.* 15*s.* 10*d.* 2*q.*
9. 16 chaldron of wheat, at 1*l.* 18*s.* 8*d.* 2*q.* per chaldron.
Ans. 30*l.* 19*s.* 4*d.* 0*q.*
10. 18 gross of filk stockings, at 7*l.* 7*s.* 6*d.* per gross.
Ans. 132*l.* 15*s.* 0*d.* 0*q.*
11. 20 cwt. of Malaga raisins, at 1*l.* 12*s.* 4*d.* per cwt.
Ans. 32*l.* 6*s.* 8*d.* 0*q.*
12. 24 chests of green tea, at 9*l.* 16*s.* 6*d.* per chest.
Ans. 235*l.* 16*s.* 0*d.* 0*q.*
13. 36 hogsheads of sugar, at 11*l.* 12*s.* 9*d.* 3*q.* per hoghead.
Ans. 419*l.* 1*s.* 3*d.* 0*q.*
14. 48 tons of hay, at 3*l.* 12*s.* 8*d.* 2*q.* per ton.
Ans. 174*l.* 10*s.* 0*d.* 0*q.*
15. 72 butts of beer, at 4*l.* 4*s.* 3*d.* per butt.
Ans. 303*l.* 6*s.* 0*d.* 0*q.*

40 COMPOUND MULTIPLICATION.

16. 96 acres of land, at 36*l.* 18*s.* 8*d.* per acre.
Ans. 3545*l.* 12*s.* 0*d.* 0*q.*
17. 120 dozen of red port, at 1*l.* 12*s.* 6*d.* per dozen.
Ans. 195*l.* 0*s.* 0*d.* 0*q.*
18. 144 loads of oak, at 3*l.* 15*s.* 6*d.* per load.
Ans. 543*l.* 12*s.* 0*d.* 0*q.*
19. 216 tons of iron, at 25*l.* 16*s.* 8*d.* 2*q.* per ton.
Ans. 5580*l.* 9*s.* 6*d.* 0*q.*
20. 1728 chaldron of coals, at 2*l.* 18*s.* 8*d.* per chaldron.
Ans. 5068*l.* 16*s.* 0*d.* 0*q.*

EXAMPLES IN WEIGHTS AND MEASURES.

1. Multiply 24*lb.* 11*oz.* 14*dwt.* 22*gr.* by 6.
Ans. 149*lb.* 10*oz.* 9*dwt.* 12*gr.*
2. Multiply 34*T.* 14*C.* 2*Q.* 22*lb.* 14*oz.* 12*dr.* by 8.
Ans. 277*T.* 17*C.* 2*Q.* 15*lb.* 6*oz.* 0*dr.*
3. Multiply 36*lb.* 11*3.* 73. 29. 16*gr.* by 12.
Ans. 443*lb.* 113. 73. 09. 12*gr.*
4. Multiply 48*last.* 11*sa.* 1*W.* 6*T.* 1*st.* 1*cl.* by 16.
Ans. 784*last.* 0*sa.* 0*W.* 4*T.* 0*st.* 0*cl.*
5. Multiply 24*D.* 59*M.* 7*F.* 36*P.* 5*yd.* 2*ft.* 9*in.* by 24.
Ans. 599*D.* 59*M.* 6*F.* 9*P.* 4½*yd.* 0*ft.* 0*in.*
6. Multiply 96*E.E.* 2*qrs.* 3*n.* 1*in.* by 35.
Ans. 2957*E.E.* 0*qrs.* 0*N.* 1½*in.*
7. Multiply 122*A.* 2*R.* 36*P.* 24*yds.* 6*ft.* 122*in.* by 45.
Ans. 5522*A.* 3*R.* 6*P.* 27*yd.* 2*ft.* 18*in.*
8. Multiply 72*tun.* 3*bhd.* 55*gal.* 7*pts.* by 63.
Ans. 4597*tun.* 0*bhd.* 55*gal.* 1*pt.*
9. Multiply 96*tun.* 2*pun.* 3*K.* 1*F.* 8*B.G.* 7*pts.* by 81.
Ans. 7855*tun.* 1*pun.* 1*K.* 0*F.* 7*B.G.* 7*pts.*
10. Multiply 124*last.* 9*qrs.* 7*bus.* 3*pc.* 1*gal.* 7*pt.* by 108.
Ans. 13499*last.* 9*qrs.* 6*bus.* 1*P.* 0*gal.* 4*pt.*
11. Multiply 21*cent.* 95*yr.* 10*mon.* 3*W.* 4*D.* 22*H.* 12*M.* 24*S.*
by 864.
Ans. 18972*cent.* 6*yrs.* 1*mon.* 3*W.* 6*D.* 10*H.* 33*M.* 36*S.*

COMPOUND DIVISION.

COMPOUND DIVISION is the mode of finding a number, such, that any two numbers, a simple and compounded, being given, the number found shall be contained by the compounded so often as there are units in the simple.

The compounded number is called the Dividend.

The

The simple number is called the Divisor.

The number found is called the Quotient; by determining which, the following problem is solved.

PROBLEM X.

Any two numbers, a simple and compounded, being given, to determine the third that shall be contained by the compounded so often as there are units in the simple.

RULE*.

1. Place the divisor and dividend as in simple division, and divide the place of the quotient into spaces with titles similar to the dividend.

2. Divide the highest denomination of the dividend by the divisor, as in simple division, and place the quotient in that space with the highest title.

3. If there be any remainder after this division, find how many units of the next lower order it is equal to (which is done by multiplying the remainder by the rate of increase at that place); and add those units to the number, if any, which stands in that order.

4. Divide the resulting number by the divisor as before, and place the quotient in the space next to the former, under its proper title: proceed in like manner through all; and the whole quotient, thus found, will be that required.

5. But if the divisor is the product of several single factors, divide the dividend by these several factors continually, and the last quotient will be that required.

The method of proof is the same in this as the first in simple division.

* Simple and Compound Division have the same relation to one another as simple and compound multiplication; and as division is only the converse of multiplication, if what has been already explained be well understood, the truth of this rule will appear evident: for in both divisions the dividend is resolved into parts out of which the divisor is successively taken, and each place in the quotient is of the same denomination with that of the dividend from whence it was taken; therefore the quotient will be contained by the dividend so often as there are units in the divisor, let the denominations be what they will.

EXAMPLES IN MONEY.

1. If 6 yards of cloth cost 5*l.* 11*s.* 3*d.* 0*q.* what is the price per yard? *Ans.* 0*l.* 18*s.* 6*d.* 2*q.*
2. If 7 cwt. of cheese cost 24*l.* 13*s.* 6*d.* 0*q.* what is the price per cwt. *Ans.* 3*l.* 10*s.* 6*d.* 0*q.*
3. If 8 firkins of butter cost 15*l.* 8*s.* 6*d.* 0*q.* what is the price per firkin? *Ans.* 1*l.* 18*s.* 6*d.* 3*q.*
4. If 9 quarters of wheat cost 11*l.* 0*s.* 10*d.* 2*q.* what is the price per quarter? *Ans.* 1*l.* 4*s.* 6*d.* 2*q.*
5. If 10 anchors of brandy cost 26*l.* 9*s.* 9*d.* 2*q.* what is the price per anchor? *Ans.* 2*l.* 12*s.* 11*d.* 3*q.*
6. If 11 hogheads of porter cost 92*l.* 13*s.* 8*d.* 3*q.* what is the price per hoghead? *Ans.* 8*l.* 8*s.* 6*d.* 1*q.*
7. If 12 tuns of wine cost 1210*l.* 18*s.* 6*d.* 0*q.* what is the price per tun? *Ans.* 100*l.* 18*s.* 2*d.* 2*q.*
8. If 14 yards of broad cloth cost 22*l.* 15*s.* 10*d.* 2*q.* what is the price per yard? *Ans.* 1*l.* 12*s.* 6*d.* 3*q.*
9. If 16 chaldron of wheat cost 30*l.* 19*s.* 4*d.* 0*q.* what is the price per chaldron? *Ans.* 1*l.* 18*s.* 8*d.* 2*q.*
10. If 18 gros of stockings cost 132*l.* 15*s.* 0*d.* 0*q.* what is the price per gros? *Ans.* 7*l.* 7*s.* 6*d.* 0*q.*
11. If 20 cwt. of Malaga raisins cost 32*l.* 6*s.* 8*d.* 0*q.* what is the price per cwt. *Ans.* 1*l.* 12*s.* 4*d.* 0*q.*
12. If 24 chests of green tea cost 235*l.* 16*s.* 0*d.* 0*q.* what is the price per chest? *Ans.* 9*l.* 16*s.* 6*d.* 0*q.*
13. If 36 hogheads of sugar cost 419*l.* 1*s.* 3*d.* 0*q.* what is the price per hoghead? *Ans.* 11*l.* 12*s.* 9*d.* 3*q.*
14. If 48 tons of hay cost 174*l.* 10*s.* 0*d.* 0*q.* what is the price per ton? *Ans.* 3*l.* 12*s.* 8*d.* 2*q.*
15. If 72 butts of beer cost 303*l.* 6*s.* 0*d.* 0*q.* what is the price per butt? *Ans.* 4*l.* 4*s.* 3*d.* 0*q.*
16. If 96 acres of land cost 3545*l.* 12*s.* 0*d.* 0*q.* what is the price per acre? *Ans.* 36*l.* 18*s.* 8*d.* 0*q.*
17. If 120 doz. of red port cost 195*l.* 0*s.* 0*d.* 0*q.* what is the price per doz. *Ans.* 1*l.* 12*s.* 6*d.* 0*q.*
18. If 140 loads of oak cost 543*l.* 12*s.* 0*d.* 0*q.* what is the price per load? *Ans.* 3*l.* 15*s.* 6*d.* 0*q.*
19. If 216 tons of iron cost 5580*l.* 9*s.* 6*d.* 0*q.* what is the price per ton? *Ans.* 25*l.* 16*s.* 8*d.* 2*q.*
20. If 1728 chaldron of coals cost 5068*l.* 16*s.* 0*d.* 0*q.* what is the price per chaldron? *Ans.* 2*l.* 18*s.* 8*d.* 0*q.*

COMPOUND DIVISION.

43

EXAMPLES IN WEIGHTS AND MEASURES.

1. Divide 149lb. 10oz. 9dwt. 12gr. by 6.
Ans. 24lb. 11oz. 14dwt. 22gr.
2. Divide 277t. 17c. 2q. 15lb. 6oz. 0dr. by 8.
Ans. 34t. 14c. 2q. 22lb. 14oz. 12dr.
3. Divide 443lb. 113. 73. 09. 12gr. by 12.
Ans. 36lb. 113. 73. 29. 16gr.
4. Divide 784last. 0sa. 0w. 4t. 0st. 0lb. by 16.
Ans. 48last. 11sa. 1w. 6t. 1st. 1lb.
5. Divide 599deg. 59mil. 6f. 9p. 4 $\frac{1}{2}$ yd. of. 0in. by 24.
Ans. 24deg. 59mil. 7f. 36p. 5yd. 2f. 9in.
6. Divide 2957Eng.ell. 0qrs. 0nl. 1 $\frac{1}{2}$ in. by 35.
Ans. 96Eng.ell. 2qr. 3nl. 1in.
7. Divide 5522ac. 3rd. 6p. 27yd. 2ft. 18in. by 45.
Ans. 122ac. 2rd. 36p. 24yd. 6ft. 12in.
8. Divide 4597tuns. 0bhd. 55gal. 1pt. by 63.
Ans. 72tun. 3bhd. 55gal. 7pt.
9. Divide 7855tuns. 1pun. 1kil. of. 7B.G. 7pt. by 81.
Ans. 96tuns. 2pun. 3kil. 1fir. 8B.G. 7pt.
10. Divide 13499last. 9qr. 6bus. 1p. 0gal. 4pt. by 108.
Ans. 124last. 9qr. 7bus. 3p. 1gal. 7pt.
11. Divide 18972cent. 6yr. 1mon. 3w. 6d. 10h. 3min. 36sec.
Ans. 21cent. 95yr. 10mon. 3w. 4d. 22h. 12min. 24sec.

REDUCTION.

REDUCTION is the mode of converting numbers from one denomination to another, without changing their values; by performing which, the following problem is solved.

PROBLEM XI.

Any number being given, to change its denomination, and retain its value.

RULE *.

- I. When the numbers are to be reduced from a higher to a lower denomination.

1. Multi-

* In reducing numbers from a higher to a lower denomination, the highest order is supposed first of all to be multiplied by its own denominator,

1. Multiply the numerator of the highest order by the denominator of the next lower, and increase the product by the numerator of the multiplier.

2. Make this increased product a multiplicand, and the denominator of the next lower order to the former a multiplier, and increase the product by its numerator as before.

3. Continue the process through all the various orders, or denominations, to the lowest, and the number last found will be that required.

II. When the numbers are to be reduced from a lower to a higher denomination.

1. Divide the number given by its own denominator, and set down the remainder.

2. Divide the quotient by the denominator of the next higher order, and set down the remainder as before.

3. Continue the process through all the various orders, or denominations, to the highest; and the quotient last found, together with the several remainders, will be that required.

The first and second parts, like multiplication and division, mutually prove each other.

EXAMPLES OF MONEY.

1. In 4360*l.* how many shillings, pence, and farthings?

Ans. 87200*s.* 1046400*d.* 4185600*q.*

2. In 2482*l.* 12*s.* how many shillings, pence, and farthings?

Ans. 49652*s.* 595824*d.* 2383296*q.*

denominator, although not mentioned: for by this the denominator is only taken away, and the figures in the numerator undergo no change.

Let $\frac{a}{m} + \frac{b}{n} + \frac{c}{u} + \frac{d}{v} =$ any number whatever to be reduced to the lowest denomination expressed; the results from the several steps of the process will appear thus:

$\frac{a}{m} \times m = a, \quad \overline{a \times n + b} = na + b, \quad \overline{na + b \times u + c} =$
 $nu a + ub + c, \quad \overline{nu a + ub + c \times v + d} = nuva + uvb +$
 $vc + d =$ the number in its lowest denomination; and by reverting the process, the number would be again reduced to $\frac{a}{m} + \frac{b}{n} + \frac{c}{u} + \frac{d}{v}$ as before.

3. In

3. In 230409rs. how many shillings and pounds?
Ans. 1920s. 96l.
4. In 552 marks, each 13s. 4d. and 89 nobles, each 6s. 8d.
how many pounds?
Ans. 395l.
5. In 399l. how many ducats, each 4s. 9d.
Ans. 3280 ducats.
6. In 304842 moidores, how many French crowns?
Ans. 1829052 Fr. crowns.
7. In 936l. how many half-guineas, crowns, half-crowns,
shillings, and six-pences, each an equal number?
Ans. 960 of each.
8. In 2239488 farthings, how many moidores, and French
crowns?
Ans. 1728 moid. and 10348 Fr. crowns.
9. 2292l. how many guineas, half-guineas, quarter-guineas,
angels, marks, and nobles, of each an equal number?
Ans. 960.
10. In 2232 marks, Scots, how many pounds sterling?
Ans. 124.
11. In 439l. 17s. 6d. 2q. how many farthings?
Ans. 422322.
12. In 661440 farthings, how many pounds?
Ans. 689l.

EXAMPLES IN WEIGHTS AND MEASURES.

1. In 44lb. 10oz. 12dwt. troy, how many grains?
Ans. 258528 gr.
2. In 2453760 grains, how many pounds, troy?
Ans. 426lb.
3. In 48t. 12c. 2q. 22lb. avoirdupois, how many drams?
Ans. 27889152dr.
4. In 55050240 drams, how many tons?
Ans. 96t.
5. In 144lb. 103. 63. 29. 16gr. apoth. how many grains?
Ans. 794656gr.
6. In 1278720 grains, how many apoth. pounds?
Ans. 222lb.
7. In 120 lasts, 9sa. 1w. 6t. 1st. how many pounds, wool
weight?
Ans. 527800lb.
8. In 3878784lb. wool weight, how many lasts?
Ans. 888 lasts.
9. In 25020mil. 6fur. 36p. 3yd. 2ft. 8in. how many barley
corns?
Ans. 4745966180.
10. In

10. In 871896960 barley-corns, how many degrees? *Ans.* 66 deg.
11. In 4755801600 barley-corns, how many degrees? *Ans.* 360 deg.
12. In 144 ells English, how many inches? *Ans.* 6480 in.
13. In 840 mil. 160 yds. how many Rhyndland feet? *Ans.* 1398960 R. feet.
14. In 1728 acres, 3r. 36p. how many square inches? *Ans.* 89630064 inches.
15. In 688 tuns, 2 bhd. 42 gal. of wine, how many pints? *Ans.* 1388352 pints.
16. In 20157984 pints, how many tuns, wine measure? *Ans.* 9999 tuns.
17. In 448 butts, 2 barrels of beer, how many pints? *Ans.* 387648.
18. In 5759424 pints of beer, how many butts? *Ans.* 6666 butts.
19. In 964 lasts, 9qr. 6busf. 3pt. of corn, how many pints? *Ans.* 4940720 pints.
20. In 44472320 pints, how many lasts? *Ans.* 8686 lasts.
21. In 64 cent. 98yr. 8mon. 2w. 4d. 22h. 36m. 56sec. how many seconds? *Ans.* 205061284800.
22. In 69426720000 seconds, how many centuries? *Ans.* 22 cent.

SIMPLE PROPORTION.

SIMPLE PROPORTION is the mode of finding a fourth proportional to three given numbers; two of which are homogeneous, and said to constitute a ratio.

The third number is homogeneous to that found, and with it constitutes a ratio equal to the former.

The four numbers are in general called terms of the proportion; the two constitutive terms being the first and second, the other given number the third, and the number required the fourth: by determining which, the following problem is solved.

PROBLEM XII.

Any three numbers, two of which are homogeneous, being given, to determine a fourth proportional.

RULE.

RULE*.

Multiply the second and third terms together, divide their product by the first, and the quotient thus found will be that required. The first and second terms are thus distinguished.

1. Multiply

* Any two quantities of the same kind, whether mathematical or physical, constitute a ratio, and may be expressed by numbers either whole, fracted, or surd; number itself being only the abstracted ratio of two homogeneous quantities, one of which is esteemed as unity. But no two heterogeneous quantities are either capable of equality, or any mathematical relation whatever; and whenever they have any such apparent mark, it is to be thus understood. Any ratio may be represented by a fraction, the numerator of which is the antecedent, and the denominator the consequent, of the ratio:

(e. g.) the ratio of a to b is represented by $\frac{a}{b}$, and the ratio of c to d by $\frac{c}{d}$; but, if any quantity be divided by a quantity of the same kind, the quotient becomes abstract number: therefore both $\frac{a}{b}$ and $\frac{c}{d}$ are abstract numbers, and as such are capable of comparison; and if these two abstract numbers are equal, then the ratio of a to b is said to be equal to that of c to d , and is thus expressed: $\frac{a}{b} \doteq \frac{c}{d}$. The

four quantities themselves are called proportionals, being two and two in the same ratio, and are generally written thus: $a : b :: c : d$. If the above equation be multiplied by bd , there results, $ad = bc$; a well known property of four proportionals; (i. e.) the rectangle of the extremes is equal to that of the means. Hence also it appears evident, that the product of the means, divided by either extreme, gives the other; which proves the truth of the rule, and method of proof.

The manner of distinguishing the first term from the second, remains to be considered.

1. That the measure of every effect should be as the efficient agency and time of action conjointly, is thus evident:—When the agency is constant, and the time variable, the effect will be as the time; and when the time is constant, and the agency variable, the effect will be as the agency; therefore when both are variable, the effect will be as the agency and time of action conjointly. (e. g.)

If the number of agents a , perform a given effect in the time d ; and the number of equipotent agents b , perform the same effect in the time x ; it is required to determine x . From the above conclusion,

$ad \doteq bx$, $\therefore \frac{ad}{b} \doteq x$, $\therefore b : a :: d : x$; therefore b is the first term,

and

1. Multiply every power or agency by the time of its continuance, if given; which product will be as the measure of its effect.

2. Multiply the component parts of every effect together, if given; which product will be as the measure of its cause.

and a the second. This is called inverse proportion, the terms of which are in common most absurdly arranged by arithmeticians thus: $a : d :: b : x$; where the quantities compared are neither homogeneous, nor the rectangle of the extremes equal to that of the means. It is certainly time that mathematicians, at least, should adhere to the trambles of truth, if all the rest of the world should wander in the paths of delusion and error.

2. That the measure of every cause should be as the component parts of its effect conjointly, is no less evident than the former: for suppose any one constant, and the other variable, the measure of the cause will always be as the variable quantity; therefore when both are variable, the cause will be as the component parts conjointly. (e. g.) If a given motive force generate, or a given resistive force destroy, the velocity V in the body Q , and in an equal time generate or destroy the velocity v in the body q ; it is required to determine v . The effects or momenta of moving bodies in the same time are as their masses and velocities conjointly; hence $QV \doteq qv \therefore \frac{QV}{q} \doteq v$,

$\therefore q : Q :: V : v$, or, the quantities of matter are reciprocally as the velocities, and vice versa, when the forces and times are the same.

3. That the product of any one cause by the effect of its correlative should be equal to that of the other, is founded upon the following principle: That the ratio of any two causes is equal to that of their effects in the same or equal times. (e. g.) If the cause a produce the effect c in a given time, and the cause b produce the effect d in the same

or equal time. From the above principle, $\frac{a}{b} \doteq \frac{c}{d}$, $\therefore ad \doteq bc$, and $a : b :: c : d$, called direct proportion, the inverse of which is $b : a :: d : c$; in both proportions, homogeneous quantities only are compared, and the rectangles of extremes and means remain equal and unvariable while the quantities themselves remain unchanged.

N. B. If any number greater than unity will divide both the constitutive terms, without remainder, let them thereby be reduced to their lowest; and if the third term is a number of different denominations, it must either be reduced to one denomination, or the operation performed by compound multiplication and division.

3. Mul-

3. Multiply every cause by the effect of its relative, if given, and vice versa, which product will be equal to that of the other two terms.

Two such numbers as express either cause or effect, action and passion, or their component parts, will either actually be found, or may rationally be supposed to exist, in all proportions whatever, whether mathematical or physical; hence, that constitutive term which is the multiplier must be the second, and the other the first.

METHOD OF PROOF.

Multiply the number found by the first term; and if the product is equal to that of the means, the work is right: or divide the product of the means by the number found; if the quotient is equal to the first term, the work is right.

PROMISCUOUS EXAMPLES.

1. If 8 yards of velvet cost 1*l.* 12*s.* what will 144 yds. cost?
Ans. 28*l.* 16*s.*
2. How many yards of velvet may be bought for 28*l.* 16*s.* when 8 yards cost 1*l.* 12*s.*?
Ans. 144 yards.
3. What will be the price of 108 yards of cambric, of which 9 yards cost 5*l.* 12*s.*?
Ans. 67*l.* 4*s.*
4. What will nine yards of cambric cost, at the rate of 67*l.* 4*s.* for 108 yards?
Ans. 5*l.* 12*s.*
5. If 14cwt. 2qr. of sugar cost 53*l.* 0*s.* 8*d.* what will 87cwt. cost?
Ans. 318*l.* 4*s.*
6. What quantity of sugar may be bought for 318*l.* 4*s.* at the rate of 14cwt. 2qr. for 53*l.* 0*s.* 8*d.*?
Ans. 87cwt.
7. If a workman gains one guinea in 6 days, how much will he gain in 123 days?
Ans. 21*l.* 10*s.* 6*d.*
8. If a workman's gain in 123 days be 21*l.* 10*s.* 6*d.* what is that per week?
Ans. 1*l.* 1*s.*
9. If 35 yards of broad cloth cost 39*l.* 7*s.* 6*d.* how many yards may be bought for 19*l.* 2*s.* 6*d.*?
Ans. 17 yards.
10. If 17 yards of broad cloth cost 19*l.* 2*s.* 6*d.* what will 35 yards of the same kind cost?
Ans. 39*l.* 7*s.* 6*d.*
11. If 6 yards of Holland cost 2*l.* 6*s.* 9*d.* what will 66 yds. 2qrs. cost?
Ans. 25*l.* 18*s.* 1*d.* 39.
12. If 66 yards 2 qrs. of Holland cost 25*l.* 18*s.* 1*d.* 39. what is the price per yard?
Ans. 0*l.* 7*s.* 9*d.* 29.
13. If 24 yards of linen cost 3*l.* 6*s.* what will 8 pieces, each 54 yards, cost?
Ans. 59*l.* 8*s.* 0*d.*

14. If 432 yards of linen cost 59*l.* 8*s.* what is the price per yard? *Ans.* 0*l.* 2*s.* 9*d.*
15. How much silver may I have for 760*l.* 10*s.* 2*d.* 1*q.* at 5*s.* 9*d.* per ounce? *Ans.* 220*lb.* 5*oz.* 5*dwt.*
16. What is the value of a piece of silver weighing 220*lb.* 5*oz.* 5*dwt.* at 5*s.* 9*d.* per ounce? *Ans.* 760*l.* 10*s.* 2*d.* 1*q.*
17. If 8 stones of wool cost 12*s.* what is the value of 12 lasts? *Ans.* 280*l.* 16*s.*
18. If 12 lasts of wool cost 280*l.* 16*s.* what is the price per stone? *Ans.* 0*l.* 1*s.* 6*d.*
19. What is the value of 64 tuns of wine, at 25*l.* 4*s.* per hogshead? *Ans.* 645*l.* 4*s.*
20. If 64 tuns of wine cost 645*l.* 4*s.* what is the price per gallon? *Ans.* 0*l.* 8*s.* per gal.
21. If 3 quarters of corn cost 4*l.* 5*s.* 6*d.* what cost 24 chaldrons? *Ans.* 136*l.* 16*s.*
22. If 24 chaldrons of corn cost 136*l.* 16*s.* what is the price per bushel? *Ans.* 0*l.* 3*s.* 6*d.* 3*q.*
23. If 8 bushels of coals cost 0*l.* 9*s.* 4*d.* what will 96 chaldrons cost? *Ans.* 201*l.* 12*s.*
24. If 5cwt. of lead cost 4*l.* 1*s.* 8*d.* what cost 60 pieces, weighing each 1cwt. 12*lb.*? *Ans.* 54*l.* 5*s.*
25. If the tax upon 2291*l.* 5*s.* be 400*l.* 19*s.* 4*d.* 2*q.* what is the rate per pound? *Ans.* 0*l.* 3*s.* 6*d.* per *£*.
26. What does the tax upon 2291*l.* 5*s.* amount to, at the rate of 3*s.* 6*d.* per *£*.? *Ans.* 400*l.* 19*s.* 4*d.* 2*q.*
27. If gold be worth 3*l.* 10*s.* per ounce, what is the value of 12 ingots, each 9*lb.* 9*oz.* 12*dwt.*? *Ans.* 4939*l.* 4*s.*
28. If 12 ingots of gold, each weighing 9*lb.* 9*oz.* 12*dwt.* be worth 4939*l.* 4*s.* what is that per grain? *Ans.* 0*l.* 0*s.* 1*d.* 3*q.*
29. How many men will perform in 4 days what 16 men can finish in 12 days? *Ans.* 48 men.
30. In how many days will 12 men finish a piece of work which 48 men can do in 4 days? *Ans.* 16 days.
31. How many yards of canvas that is ell wide will line 40 yards of say that is 3 quarters wide? *Ans.* 24 yards.
32. If 24cwt. is carried 36 miles for a given sum, how far may 3 tons 12cwt. be carried for the same? *Ans.* 12 miles.
33. If A. lends B. 650*l.* for 22 months, how long ought B. to lend A. 953*l.* 6*s.* 8*d.* to requite his kindness? *Ans.* 15 months.

34. If

term
are f

34. If I travel a given distance in 12 days of 8 hours each, in how many days of 12 hours each will I do the same?
Ans. 8 days.
35. If, when wheat is 6s. per bushel, the penny loaf weighs 9oz. what ought it to weigh when the bushel is 8s.?
Ans. 6oz. 12dr.
36. If, when wine is 8s. per gallon, I drink 20 pints per week, how much may I drink per week, without extra expence, when the wine is at 10s. per gallon?
Ans. 16 pints.
37. A garrison, being besieged, has in it 6 months provisions, at the rate of 12 ounces per day for each man; but hearing that it cannot be relieved till after 9 months, how much must each man have per day, that the provisions may last that time?
Ans. 8 ounces.
38. How many yards of matting, 3 feet broad, will cover a room floor that is 12 yards long and 7 yards wide?
Ans. 84 yards.
39. If 22l. worth of oats serve 56 horses for a given time, when the oats are at 12s. per boll, how many horses will eat to the same value, in an equal time, when the oats are at 16s. per boll?
Ans. 42 horses.
40. A governor of a fort having provisions sufficient to serve 2820 men for 6 months, how many of them must he dismiss that the provisions may serve 4 months longer?
Ans. 1128 men.
41. Bought 8 bales of cloth, each containing 6 pieces, and each piece 28 yards, at 16l. 16s. per piece, what is the value of the whole, and the rate per yard?
Ans. 806l. 8s. at 12s. per yard.
42. If a gentleman's yearly income is 12,480l. how much may he spend per day to save 4121l. 10s. per annum?
Ans. 22l. 18s. per day.

EXAMPLES OF LOSS AND GAIN*.

1. How must I sell tea per lb. that cost me 12s. to gain after the rate of 25 per cent.
Ans. 15s. per lb.
2. If

* Questions of this kind are such, the solutions of which determine the loss or gain in the various transactions of trade, and are solved by simple proportion.

2. If a pound of sugar is bought for 8*d.* and sold for 10*d.* what is the gain per cent. ? *Ans.* 25 per cent.

Although the lame invectives thrown out by some scribblers against Doctor Hutton's very judicious remark upon some questions of this kind, are truly below all censure; nevertheless I hope it will be acceptable to some, whose eyes are not blinded with prejudice, to see the proposition demonstrated.

Dem. Let m = the prime cost of any given quantity of goods whatever, x and z = two several gains or losses upon the same, and $c = 100$. Then $m \pm x$ and $m \pm z$ = the two selling prices, = a and n if given,

$$\text{and } \left\{ \begin{array}{l} m : x :: c : \frac{cx}{m} = r \\ m : z :: c : \frac{cz}{m} = s \end{array} \right\} = \text{the rates of gain or loss per cent.}$$

$$\text{But } m \pm x : m \pm z :: c \frac{m \pm x}{m} : c \frac{m \pm z}{m} \therefore \text{universally, } a : n :: c \pm r : c \pm s. \quad Q. E. D.$$

The several authors mentioned by Mr. Hutton, who have brought out false solutions, found their calculations upon the following false analogy; viz. $m \pm x : m \pm z :: \frac{cx}{m} : \frac{cz}{m}$, which, turned to an equation, and reduced, gives $x = z$ contra hypothesis.

N. B. I deem it altogether unnecessary to give any examples of Tare and Tret; any person, with the use of common sense, and compound subtraction, being equal to the task. Gross weight of any commodity is its own weight, together with that of its package, whether cask, chest, &c.

Tare is the weight of the package, or an allowance made instead of it. — What remains, after the tare is taken from the gross, may be called tare futtle, if there be more deductions.

Tret is an allowance of 4 lb. upon every 104 lb. of tare futtle, on account of dust or other waste. What remains, after tret is deducted, may be called tret futtle, if there be more deductions. Cloff is an allowance of 2 lb. for every 3 cwt. and some say for every cwt. of tret futtle, to make the weight hold good when sold by retail.

When all the deductions are made, the last remainder is called neat weight.

3. Bought

3. Bought hops at 4*l*. 16*s*. and sold them for 4*l*. what was the loss per cent.?
Ans. 16*l*. 13*s*. 4*d*. per cent.
4. Bought 17*cw*. 2*q*. 14*lb*. of cheese at 1*l*. 12*s*. per cwt. and sold it for 2*l*. 12*s*. per cwt. what was gained upon the whole?
Ans. 17*l*. 12*s*. 6*d*.
5. If 17*cw*. 2*q*. 14*lb*. be bought for 28*l*. 4*s*. and sold for 45*l*. 16*s*. 6*d*. what is the rate of gain per cwt.?
Ans. 1*l*. per cwt.
6. Bought 12*cw*. 2*q*. 14*lb*. for 70*l*. 14*s*. how must I sell it per lb. to gain 25 per cent.?
Ans. 1*s*. 3*d*. per lb.
7. If, when I sell cloth at 8*s*. per yard, I gain 10 per cent. what will be the gain per cent. when it is sold for 10*s*. 6*d*. per yard?
Ans. 44*l*. 7*s*. 6*d*. per cent.
8. If, when I sell sugar at 8*s*. per stone, I lose 6*l*. per cent. what do I gain or lose per cent. when I sell it at 10*s*. per stone?
Ans. 17*l*. 10*s*. gain per cent.
9. If I buy 28 pieces of stuff, at 4*l*. per piece, and sell 10 of the pieces at 6*l*. and 8 at 5*l*. per piece; at what rate per piece must I sell the remaining 10, to gain 20 per cent. by the whole?
Ans. 3*l*. 8*s*. 9*d*. 2*q*. $\frac{2}{3}$ per piece.
10. Having sold 12 yards of cloth for 5*l*. 8*s*. and thereby gained 8*l*. per cent. what was the prime cost per yard?
Ans. 8*s*. 4*d*. per yard.
11. If, by selling fustians at 11*s*. 6*d*. per end, I gain 15*l*. per cent. what do I gain per cent. by selling the same quantity at 12*s*.?
Ans. 20*l*. per cent.
12. Bought 48 gallons of brandy at 4*s*. per gallon, 8 of which were lost by accident; at what rate per gallon must I sell the remainder, to gain 25*s*. per cent. upon the whole prime cost?
Ans. 6*s*. per gallon.

COMPOUND PROPORTION.

COMPOUND PROPORTION is the mode of finding a number the ratio of which to a number given shall be equal to that compounded of two, three, or more given ratios.

The rules given for simple proportion are universal, and extend to all proportions, however complicated. From a proper consideration of these principles, the following rule is derived.

REG.

RULE.

RULE*.

1. Arrange the terms of supposition in one line, so that all the component parts of the cause may succeed one another from the left toward the right, and all the component parts of the effect follow in their order.

2. Arrange the terms of demand in a line underneath the former, so that homogeneous terms may stand directly under each other, and mark the place of the unknown quantity with Q or X .

3. Multiply all the producing terms in the one line into all the produced in the other continually; and the two products will form an equation.

4. Make the continual product of the known terms, on that side of the equation where the unknown quantity is concerned, a divisor; and the like product of all the terms, on the other side, a dividend: the quotient resulting from this division will be that required.

The method of proof is the same in this as in simple proportion.

PROMISCUOUS EXAMPLES.

1. If 12 men in 16 days perform 96 roods of work, in what time will 8 men perform 120 roods? *Ans. 30 days.*

* The truth of this rule will appear evident by applying the general principles of proportion already explained to the following equation:

$$\frac{S}{s} \div \frac{V}{v} \times \frac{T}{t} \times \frac{D}{d} \times \frac{N}{n} \times \frac{r}{R};$$

that is—the spaces to which spheres of different diameters and densities, with different velocities and times of motion, will penetrate into substances of different resistive powers, are as their diameters, densities, velocities, and times of motion directly, and as the resistive powers of the substances penetrated inversely. Suppose now that S was required, and all the rest given; we have from the above equation

$$S \div \frac{V}{v} \times \frac{T}{t} \times \frac{D}{d} \times \frac{N}{n} \times \frac{r}{R} \times \frac{s}{1};$$

but from the first and second parts of the rule, the terms would be thus arranged: $\frac{v, t, d, n, r, s}{V, T, D, N, R, S}$, and

by the third $v \times t \times d \times n \times R \times S = V \times T \times D \times N \times r \times s$,

and by the fourth $S = \frac{V \times T \times D \times N \times r \times s}{v \times t \times d \times n \times R \times 1}$, the same expression as before.

COMPOUND PROPORTION.

55

2. If 12 men in 18 days gain 37*l.* 16*s.* how many men will gain 378*l.* in 24 days? *Ans.* 90 men.
3. If 16 horses in 56 days eat 120 bushels of corn, in what time will 24 horses eat 18 lasts? *Ans.* 448 days.
4. If 15 men in 6 days eat 13*s.* worth of bread, when wheat is at 12*s.* per boll, in what time will 60 men eat to the amount of 4*l.* 6*s.* 8*d.* when wheat is at 10*s.* per boll? *Ans.* 12 days.
5. If 4 men in 16 days, working 15 hours per day, clear 18 acres of ground, how much will be done by 8 men in 4 days, working 10 hours per day? *Ans.* 6 acres.
6. If, when corn is at 4*s.* per bushel, 8 ounces of bread be worth 2*d.* what is the value of 30 ounces, when corn is at 6*s.* per bushel? *Ans.* 11*d.* 1*q.*
7. If the carriage of 12*cw.* 1*q.* for 72 miles be 2*l.* 12*s.* 6*d.* what will be the carriage of 1 ton, 16*cw.* 3*q.* for 144 miles? *Ans.* 15*l.* 14*s.*
8. If 4000*lb.* of beef serve 250 seamen 16 days, what quantity of beef will serve 2000 seamen 48 days? *Ans.* 96,000*lb.*
9. If 12 men build a wall 30 feet long, 6 feet high, and 3 feet thick, in 15 days, working 10 hours per day; in how many days, 12 hours each, will 60 men build a wall 300 feet long, 8 feet high, and 6 feet thick? *Ans.* 66 days, 8 hours.
10. If a bar of iron 5 feet 4 inches long, 7 inches broad, and 4 inches thick, weigh 4*cwt.* what weight of mercury would that tube contain, the internal dimensions of which were each double those of the bar, the specific gravity of mercury to that of iron being as 2 : 1? *Ans.* 3 ton. 4*cwt.*

EXAMPLES OF SIMPLE INTEREST.

1. If 100*l.* in one year gain 5*l.* what will be the interest of 750*l.* for 12 years? *Ans.* 450*l.*
2. If 750*l.* in 12 years gain 450*l.* in what time will 1206*l.* gain 1036*l.* 16*s.* *Ans.* 16 years.
3. If 100*l.* in one year gain 4*l.* what will be the interest of 720*l.* for 12 years? *Ans.* 345*l.* 12*s.*
4. If 720*l.* in 8 years gain 230*l.* 8*s.* what will 2160*l.* gain in 16 years? *Ans.* 1382*l.* 8*s.*
5. If 880*l.* in 7 years gain 308*l.* what is the rate per cent. *Ans.* 5*l.* per cent.
6. If

6. If 960*l.* in 16 years gain 921*l.* 12*s.* what is the rate per cent.? *Ans.* 6*l.* per cent.
7. If 100*l.* in one year gain 2*l.* 10*s.* what will 808*l.* gain in 8 years? *Ans.* 161*l.* 12*s.*
8. If 808*l.* in 4 years gain 80*l.* 16*s.* what is the rate per cent.? *Ans.* 2*l.* 10*s.* per cent.
9. If 242*l.* 10*s.* gain 130*l.* 19*s.* in 12 years, what is the rate per cent.? *Ans.* 4*l.* 10*s.* per cent.
10. If 100*l.* in one year gain 4*l.* 10*s.* what will 228*l.* gain in 10 years? *Ans.* 102*l.* 12*s.*
11. If 660*l.* in 12 years and a quarter gain 282*l.* 19*s.* 6*d.* what is the rate per cent.? *Ans.* 3*l.* 10*s.* per cent.
12. What will 660*l.* gain in 12 years and a half, at 3*l.* 10*s.* per cent.? *Ans.* 288*l.* 15*s.*

EXAMPLES OF COMMISSION.

1. What comes the commission of 500*l.* 13*s.* 4*d.* to, at 3*l.* per cent.? *Ans.* 15*l.* 0*s.* 4*d.* 3*q.* $\frac{1}{2}$
2. What must I allow my correspondent for disburſing on my account 3774*l.* at $2\frac{1}{2}$ per cent.? *Ans.* 94*l.* 7*s.*
3. If I allow my factor $4\frac{1}{2}$ per cent. what may he demand on laying out 12,000*l.*? *Ans.* 540*l.*
4. If my factor demands 540*l.* for laying out 12,000*l.* at what rate percent. is his commission? *Ans.* $4\frac{1}{2}$ per cent.

EXAMPLES OF BROKERAGE.

1. If I allow my broker 2*l.* 10*s.* per cent. what may he demand when he ſells goods to the value of 876*l.* 10*s.*? *Ans.* 21*l.* 18*s.* 3*d.*
2. If my broker ſells goods to the amount of 660*l.* what is his demand at $1\frac{1}{2}$ per cent.? *Ans.* 9*l.* 18*s.*
3. If a broker ſells goods to the amount of 1000 guineas, what is his due at 2*l.* per cent.? *Ans.* 21*l.*
4. If my broker demands 42*l.* for ſelling goods to the amount of 2000 guineas, what is his charge per cent.? *Ans.* 2*l.* per cent.

EXAMPLES OF INSURANCE.

1. What is the insurance of 9000*l.* at 10*l.* 15*s.* per cent.? *Ans.* 967*l.* 10*s.*
2. What is the insurance of 12,000*l.* at 8*l.* 10*s.* per cent.? *Ans.* 1020*l.*
3. What

3. What is the insurance of 874*l.* 14*s.* 2*d.* at 12*l.* 10*s.* per cent.?
Ans. 109*l.* 6*s.* 9*d.* 1*q.*
4. If the insurance of 874*l.* 14*s.* be 109*l.* 6*s.* 9*d.* 1*q.* what is the rate per cent.?
Ans. 12*l.* 10*s.* per cent.

PRACTICE.

PRACTICE is a name which arithmeticians have been pleased to give to a few concise methods of solving questions in simple proportion, when one of the constitutive terms is unity: hence very many questions of this kind may be solved either by compound multiplication or division; but when this cannot conveniently be done, they are commonly solved by the help of measures or aliquot parts, as the following rules direct.

TABLE OF THE ALIQUOT PARTS OF MONEY.

S.	D.	£.	S.	D.	£.	S.	D.	£.	S.
10	0	$\frac{1}{2}$	1	3	$\frac{1}{18}$	3		$\frac{1}{10}$	$\frac{1}{4}$
6	8	$\frac{1}{3}$	1	0	$\frac{1}{20}$	2	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{5}$
5	0	$\frac{1}{4}$	10		$\frac{1}{10}$	2		$\frac{1}{10}$	$\frac{1}{5}$
4	0	$\frac{1}{5}$	8		$\frac{1}{8}$	1	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{5}$
3	4	$\frac{1}{6}$	7	$\frac{1}{2}$	$\frac{1}{12}$	1	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{5}$
2	6	$\frac{1}{8}$	6		$\frac{1}{6}$	1		$\frac{1}{10}$	$\frac{1}{5}$
2	0	$\frac{1}{10}$	5		$\frac{1}{5}$	$\frac{3}{4}$		$\frac{1}{10}$	$\frac{1}{5}$
1	8	$\frac{1}{12}$	4		$\frac{1}{4}$	$\frac{1}{2}$		$\frac{1}{10}$	$\frac{1}{5}$
1	4	$\frac{1}{15}$	3	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{4}$		$\frac{1}{10}$	$\frac{1}{5}$

TABLE OF THE ALIQUOT PARTS OF A CWT.

lb.	cwt.	gr.	lb.	cwt.	gr.
56		$\frac{1}{2}$	7		$\frac{1}{8}$
28		$\frac{1}{4}$	4		$\frac{1}{16}$
16		$\frac{1}{7}$	3	$\frac{1}{2}$	$\frac{1}{8}$
14		$\frac{1}{8}$	2		$\frac{1}{16}$
8		$\frac{1}{14}$	1	$\frac{1}{4}$	$\frac{1}{8}$

RULE I.

If the given price of the integer be an aliquot part of a penny, shilling, or pound take the same part of the given quantity, the price of which is to be found, for the answer in pence, shillings, or pounds respectively.

EXAMPLES.

EXAMPLES.

Questions.				Answers.			
		s.	d.		£.	s.	d.
2185	at	10	0	_____	1092	10	0
1430	at	6	8	_____	476	13	4
2992	at	5	0	_____	748	00	0
1260	at	4	0	_____	252	0	0
2493	at	3	4	_____	415	10	0
550	at	2	6	_____	68	15	0
1974	at	2	0	_____	197	8	0
7780	at	1	8	_____	648	6	8
1872	at	1	4	_____	124	16	0
318	at	1	3	_____	19	17	6
1888	at	1	0	_____	94	8	0
2605	at	0	10	_____	108	10	10
752	at	0	8	_____	25	1	4
2748	at	0	$7\frac{1}{2}$	_____	85	17	6
2540	at	0	6	_____	63	10	0
2505	at	0	5	_____	52	3	9
1218	at	0	4	_____	20	6	0
2324	at	0	$3\frac{3}{4}$	_____	36	6	3
5012	at	0	3	_____	62	13	0
3906	at	0	$2\frac{1}{2}$	_____	40	13	9
5649	at	0	2	_____	47	1	6
4610	at	0	$1\frac{1}{2}$	_____	28	16	3
12076	at	0	$1\frac{1}{4}$	_____	62	17	11
9436	at	0	1	_____	39	6	4
7708	at	0	$0\frac{3}{4}$	_____	24	1	9
5928	at	0	$0\frac{1}{2}$	_____	12	7	0
11608	at	0	$0\frac{1}{4}$	_____	12	1	10

RULE II.

If the given price be no aliquot part of a penny, shilling, or pound; divide it into several aliquot parts, then work for each by rule 1st, and their sum will be the answer required. Or, if it can be so divided that the less become aliquot parts of the greater, then take the same parts of the prices found for the greater, and their sum will be that required as before.

EXAMPLES.

PRACTICE.

39

EXAMPLES.

Questions.				Answers.		
		s.	d.	£.	s.	d.
2440	at	11	9½	1436	0	10
3328	at	10	5½	1740	5	4
1072	at	8	3¼	443	6	4
3468	at	6	10½	1192	2	6
2802	at	4	3	595	8	6
2912	at	2	5	351	17	4
12978	at	1	7	1027	8	6
2604	at	0	11½	124	15	6
21564	at	0	11¼	1010	16	3
1820	at	0	11	83	8	5
1092	at	0	10½	47	15	6
1260	at	0	9½	49	17	6
5271	at	0	9	197	13	3
2484	at	0	8½	87	19	6
2980	at	0	8¼	102	8	9
1100	at	0	7¼	33	4	7
1084	at	0	6½	29	7	2
3960	at	0	6¼	103	2	6
956	at	0	5½	22	18	1
2412	at	0	5½	55	5	6
2468	at	0	4½	46	5	6
4768	at	0	3½	69	10	8
7164	at	0	2½	67	3	3
8692	at	0	1½	63	7	7

RULE III.

If there be pounds in the price, multiply the given quantity by their number, and take parts as before directed for the odd money, if any; which add to the former product, and the sum will be that required.

EXAMPLES.

Questions.					Answers.				
		£.	s.	d.		£.	s.	d.	
6858	at	18	10	0	_____	126873	0	0	
1415	at	5	5	0	_____	7428	15	0	
2486	at	3	13	4	_____	9115	6	8	
8527	at	2	7	6	_____	20251	12	6	
2522	at	4	3	4	_____	10508	6	8	
998	at	12	12	6	_____	12599	15	0	
					Questions.				

Questions.

Questions.					Answers.		
		£.	s.	d.	£.	s.	d.
554	at	6	18	8	3841	1	4
6485	at	7	8	10	48259	4	2
2866	at	8	15	6	25149	3	0
5862	at	9	19	10	58571	3	0

R U L E IV.

If the quantity given be a compounded number, multiply the price by the number of integers, if any, and divide the odd quantity into aliquot parts of the integer, or of each other; then take the same parts of the price of the integer, or of the price of each other; add them all to the former product, and the sum total will be that required.

E X A M P L E S.

Questions.					Answers.		
		£.	s.	d.	£.	s.	d.
18 ton, 5cwt. 2qr.	at	8	10	0 per ton	155	6	9
12 ton, 10cwt. 2qr.	at	6	6	8	79	6	6
8 ton, 15cwt. 3qr.	at	12	16	8	112	15	5½
12cwt. 2qr. 14lb.	at	16	10	6 per cwt.	208	12	6½
144 yards, 2qr.	at	2	12	9 per yard	381	2	4½
24lb. 6oz.	at	4	16	10 per pound	118	12	5
64 acres, 2r. 28p.	at	3	12	0 per acre	232	16	7
103 acres, 1r. 35p.	at	5	5	8 per acre	546	13	2½
36 hogsheads, 42gal.	at	25	4	0 per hhd.	924	0	0
42 gallons, 2 pints	at	37	16	0 per hhd.	25	7	0

R U L E V.

If the price be any even number of shillings, multiply the quantity given by half their number, doubling the first figure of the product for shillings; the rest are pounds: but if the price is any odd number of shillings, subtract one, and multiply by half the remainder as before, adding one twentieth of the given quantity for the odd shilling.

E X A M P L E S.

Questions.					Answers.	
		s.			£.	s.
675	at	4	_____		135	0
8249	at	6	_____		2474	14
7985	at	8	_____		3194	0
			2			

Questions.

Questions.

Answers.

		s.		£.	s.
998	at	12	_____	598	16
98796	at	16	_____	79036	16
18423	at	18	_____	16580	14
1464	at	7	_____	512	8
6547	at	11	_____	3600	17
17568	at	15	_____	13176	0
85674	at	17	_____	72822	18

RULE VI:

If there be a fraction in the given quantity, find the value of the integral part by the former rules; then multiply the price by the numerator of the fraction, and divide the product by the denominator; add the quotient to the value of the integral part, and the sum will be that required.

EXAMPLES.

Questions.

Answers.

		£.	s.	d.		£.	s.	d.
546 $\frac{1}{2}$	at	0	2	6	_____	68	6	3
2254 $\frac{1}{2}$	at	2	17	10	_____	6519	5	3
2653 $\frac{3}{4}$	at	0	14	0	_____	1857	12	6
1070 $\frac{1}{4}$	at	0	17	0	_____	909	14	3
788 $\frac{1}{2}$	at	12	6	8	_____	9724	16	8
659 $\frac{5}{8}$	at	6	4	10	_____	4117	3	$2\frac{1}{4}$
242 $\frac{3}{4}$	at	4	10	6	_____	1098	1	4

A R I T H M E T I C.

P A R T II.

THE DOCTRINE OF FRACTIONS.

INTEGERS and Fractions are both commensurable to unity, and best distinguished by their different functions.

Integers are numbers which express the ratio of any whole to its homogeneous part, assumed ad libitum.

Fractions are numbers which express the ratio of any part to its homogeneous whole, assumed in like manner.

The quantity assumed, or common consequent, to which the measures of both are referred, may be esteemed as unity, varied at pleasure, and subdivided without end.

The denominators, or conominals, of integers are the pure powers of ten with positive indices.

The denominators of fractions are either the pure powers of ten with negative indices, or composed of these and some other submultiples of unity.

The first kind are called Decimal Fractions; the denominators of which, like those of integers, are not written.

The second kind are called Vulgar Fractions; the reciprocals of their denominators are obliged to be written.

These reciprocals are commonly, although perhaps improperly, termed their denominators: for the proper denominators of all numbers, whether whole or fracted, are those which read in conjunction with their numerators express their proper values; but their reciprocals are numbers which have the same ratio to unity as the numerators have to their proper local values; (e. g.) $\frac{3}{4}$ is a number expressing the ratio of 3 : 4, the proper denominator of which is $\frac{1}{4}$; because that being read in conjunction with the numerator 3, expresses its proper value (i. e.) three-fourth parts = $\frac{3}{4}$; but its reciprocal (4) has the same ratio to unity as the numerator (3) to the proper value of the number ($\frac{3}{4}$); thus, 4 : 1 :: 3 : $\frac{3}{4}$.

Notwithstanding the above distinction, I shall, in the sequel of this, call the antecedents and consequents of ratios, numerators and denominators respectively; the choice of terms being in a great measure arbitrary.

The consequents of ratios, which we shall now call denominators, express the number of parts into which the unit is supposed to be divided; and the antecedents, or numerators, how many of those parts are to be taken. Integers and decimals, forming one continued chain of numbers, extending both ways, ad infinitum, might with great propriety have been explained conjointly in all the preceding modes of operation; but that I might comply with custom, that universal tyrant, and indulge the pretended weakness of juvenile faculties, I have only thus explained them in notation, confining myself to integers in the sequel of the first part.

In this second part I shall explain Vulgar and Decimal Fractions, with the theory of circulating numbers, shew their connexions, and apply them to some things not considered in the first part.

VULGAR FRACTIONS.

VULGAR FRACTIONS are divided into proper, improper, mixed, simple, compounded, and complex.

1. Proper fractions have their numerators less than their denominators, as $\frac{3}{4}$, $\frac{5}{8}$, &c.

2. Improper fractions have their numerators equal to, or greater than, their denominators, as $\frac{4}{4}$, $\frac{12}{8}$, &c.

3. Mixed fractions, or numbers, are those compounded of numbers and fractions, as $7\frac{5}{8}$, $12\frac{3}{4}$, &c.

4. Simple fractions are expressions for parts of given units, as $\frac{4}{9}$, $\frac{5}{8}$, &c.

5. Compounded fractions are expressions for the parts of given fractions, as $\frac{3}{4}$ of $\frac{5}{8}$, $\frac{2}{3}$ of $\frac{7}{11}$, &c.

6. Complex fractions have either one or both terms mixed numbers, as $\left(\frac{5\frac{2}{3}}{24}\right)$, $\frac{12}{14\frac{3}{4}}$, $\left(\frac{6\frac{5}{8}}{12\frac{2}{3}}\right)$, &c.

7. Any number which will divide two or more numbers without remainder, is called their common measure.

8. Any number which can be measured by two or more numbers, is called their common multiple, or perhaps, more properly, their common dividend.

To find the common measure a maximum, and the common multiple a minimum, are problems of considerable use in the doctrine of fractions, and are therefore solved in the first place.

PROBLEM I.

Any multitude of numbers being given, to determine their greatest common measure.

RULE*.

1. Take any two of the given numbers, and divide the greater by the less, and the divisor by the remainder, and so on, until nothing remains; the last divisor will be the greatest common measure of these two numbers.

* Let a and b be two numbers, the greatest common measure of which is required, and let them be ordered as per rule; then

$$\frac{a}{b} = c, \text{ with } d \text{ remainder.}$$

$$\frac{b}{d} = e, \text{ with } m \text{ remainder.}$$

$$\frac{d}{m} = n, \text{ with } r \text{ remainder.}$$

$$\frac{m}{r} = v, \text{ with } o \text{ remainder. I say that } r \text{ measures both } a \text{ and } b: \text{ for}$$

since r measures m , it must also measure $mn + r = d$, $de + m = b$, and $bc + d = a$; therefore r measures both a and b . I say r is also the greatest measure common to a and b ; for suppose u greater than r , to measure both a and b ; then, since u measures a , it must also measure its equal $bc + d$; but u measures b ; therefore it must also measure bc , and consequently d ; but if it measures d , it must measure de ; and it measures $de + m = b$; therefore it measures m ; but if it measures m , it must measure mn ; and it measures $mn + r = d$; hence u must also measure r , the greater the less, which is impossible: therefore r is the greatest common measure of a and b , Q. E. D. Again, if there be any multitude of numbers a, b, c, d , &c. and if the greatest common measure of a and b be found $= r$, that of r and $c = u$, and that of u and $d = v$, and so on. I say the number last found will be the greatest common measure of a, b, c, d , &c. for if this be denied, let w greater than v measure a, b, c, d . Then, since w measures a and b , it must also measure r ; but w measures c ; therefore it must also measure u ; but w likewise measures d ; therefore it also measures v : the greater the less, which is absurd; hence no number greater than v will measure a, b, c, d ; and the same mode of reasoning may be applied to any multitude of numbers whatever: therefore if there be any multitude of numbers, a, b, c, d , &c. Q. E. D.

2. Find

2. Find the greatest common measure of that found, and another of the given number as before, and so on until the whole is finished; the measure last found will be that required.

N.B. If unity is found to be the greatest common measure, the numbers are said to be prime to each other, per Euclid, lib. 7th, defin. 12th. But that gentleman is either mistaken, who affirms that they are usually called incommensurable, or they are most miserably mistaken who call them so: for magnitudes, or numbers representing them, are only said to be incommensurable, to which no common measure whatever can be assigned.

EXAMPLES.

1. What is the greatest common measure of 4165, 686, and 588? *Ans.* 49.
2. What is the greatest common measure of 1998, 918, and 522? *Ans.* 18.
3. What is the greatest common measure of 3132, 5508, and 11988? *Ans.* 108.
4. What is the greatest common measure of 24990, 4116, and 3528? *Ans.* 294.
5. What is the greatest common measure of 16660, 2744, and 2352? *Ans.* 196.
6. What is the greatest common measure of 33320, 5488, and 4704? *Ans.* 392.
7. What is the greatest common measure of 15984, 7344, and 4176? *Ans.* 144.
8. What is the greatest common measure of 8352, 14688, and 31968? *Ans.* 288.
9. What is the greatest common measure of 6264, 11016, and 23976? *Ans.* 216.
10. What is the greatest common measure of 20825, 3430, and 2940? *Ans.* 245.
11. What is the greatest common measure of 13986, 6426, and 3654? *Ans.* 126.
12. What is the greatest common measure of 29155, 4802, and 4116? *Ans.* 343.

PROBLEM II.

Any multitude of numbers being given, to determine their least common multiple.

G 3.

RULE:

R U L E *.

1. Divide the product of any two of the given numbers by their greatest common measure, and the quotient will be the least common multiple, or dividend of these two numbers.

2. Find

* The least possible dividend, common to any two numbers, is = their product divided by their greatest common measure; or, which is the same thing, = the continual product of their greatest common measure, and the least two numbers in the same proportion with those given.

DEMON. Let the two given numbers be a, b , and their greatest common measure = m ; also let u and v be the least in the same proportion with a and b . Then, $a = um$, and $b = vm$; also $\frac{ab}{m} = muv$. I say that $\frac{ab}{m} = muv$ is the least common dividend of a and b : for if this be denied, let M less than $\frac{ab}{m} = muv$, be that required. Suppose $\frac{M}{a} = d$, and $\frac{M}{b} = b$; then $M = ad = b$ $\therefore a :$

$b :: b : d :: u : v$; but since u and v are prime to each other, no numbers can be found in the same proportion, but what are some multiples of u and v : therefore u measures b , and v measures d ; but $v : d :: av : ad$; and since v measures d ; therefore also av measures ad ; but $av = \frac{ab}{m}$, and $ad = M$; hence it appears that $\frac{ab}{m}$ measures $ad = M$; that is, the greater measures the less, which is absurd: therefore M is not the least common dividend of a and b ; and in the same manner we prove that no other number but that expressed by $\frac{ab}{m} = muv$ is the least common dividend of a and b : therefore the least common dividend of any two numbers is = their product divided by their greatest common measure. Q. E. D.

Again, if there be any multitude of numbers, a, b, c, d , &c. and the least common dividend be found to any two $a, b = \frac{ab}{m}$ and the least to that and c , and so on: the last dividend found will be the least which can be measured by a, b, c, d , &c.

DEMON. Let N be the least number which can be measured by $\frac{ab}{m}$ and c . I say, it is also the least which can be measured by a, b, c : for if this be denied, let n less than N be that required. Then, since

2. Find the least common multiple of that found and another of the given numbers, and so on until the whole is finished; the number last found will be that required.

From which the following Rule may be derived.

1. Divide by any number that will measure two or more of the given numbers without remainder, and set the quotients, together with the undivided numbers, in a line below.

2. Divide the second line as before, and so on, until there are no two numbers that can be divided; then the continued product of the divisors, quotients, and undivided numbers, if any, will be that required.

EXAMPLES.

1. What is the least common multiple of 12, 9, 8, 6, 4, and 3?
Ans. 72.
2. What is the least common multiple of 6, 10, 12, 16, 18, and 20?
Ans. 720.
3. What is the least common multiple of 9, 16, 21, 14, and 28?
Ans. 1008.
4. What is the least common multiple of 36, 27, 24, 18, 12, and 9?
Ans. 216.
5. What is the least common multiple of 48, 36, 32, 24, 16, and 12?
Ans. 288.
6. What is the least common multiple of 45, 30, 15, 10, 6, and 5?
Ans. 90.
7. What is the least common multiple of 32, 24, 16, 12, 4, and 2?
Ans. 96.

since $\frac{ab}{m}$ is the least number which can be measured by a and b ; a and b can measure no number which is not some multiple of $\frac{ab}{m}$; but a, b , measure n : therefore also $\frac{ab}{m}$ measures n . And since N is the least number which can be measured by $\frac{ab}{m}$ and c ; $\frac{ab}{m}$ and c can measure no number which is not some multiple of N ; but $\frac{ab}{m}$ and c measure n ; hence also N measures n , the greater the less, which is absurd: therefore no number less than N can be measured by a, b, c ; and the same mode of reasoning may be extended to any multitude of numbers whatever: therefore if there be any multitude of numbers $a, b, c, d, \&c.$ Q. E. D.

8. What

8. What is the least common multiple of 96, 72, 48, 36, 12, and 6? *Ans.* 288.
 9. What is the least common multiple of 78, 65, 52, 39, 26, and 13? *Ans.* 780.
 10. What is the least common multiple of 9, 8, 7, 6, 5, 4, 3, 2, and 1? *Ans.* 2520.
 11. What is the least common multiple of 18, 16, 14, 12, 10, 8, 6, 4, and 2? *Ans.* 5040.
 12. What is the least common multiple of 108, 96, 84, 72, and 60? *Ans.* 15120.

REDUCTION OF VULGAR FRACTIONS.

REDUCTION OF VULGAR FRACTIONS is the mode of transforming them from one denomination to another, in order to prepare them for addition, subtraction, multiplication, and division; by performing which the following problems are solved.

PROBLEM I.

Any fraction being given, to reduce it to its lowest terms.

RULE*.

Find the greatest common measure of the two terms, by which divide both, and the respective quotients will be the terms of the fraction required.

EXAM-

* That this rule is universal, appears from the foregoing notes: for if any two numbers be divided by their greatest common measure, the quotients are the least numbers in the same proportion.

But fractions may often either be reduced to their least terms, or very much abbreviated with less trouble: for if the terms of a fraction be divided by any number which measures both without remainder; it is plain that the resulting terms, or quotients, will be in the same proportion with the first terms. Because, if the terms of any ratio be either both multiplied or both divided by the same number, it is the same as adding or subtracting the ratio of equality, which neither increases nor diminishes the value of that ratio, to or from which it is either added or subtracted.

The following Theorems are useful for abbreviating Vulgar Fractions:

THEOREMS.

1. If any number terminates on the right hand with a cypher, or a digit divisible by 2, the whole is divisible by 2; for the one which

EXAMPLES.

1. Reduce $\frac{25}{344}$ to its lowest terms. *Ans.* $\frac{5}{17}$.
2. Reduce $\frac{144}{80}$ to its lowest terms. *Ans.* $\frac{3}{10}$.
3. Reduce $\frac{52}{72}$ to its lowest terms. *Ans.* $\frac{1}{3}$.
4. Reduce $\frac{426}{729}$ to its lowest terms. *Ans.* $\frac{2}{3}$.
5. Reduce $\frac{728}{729}$ to its lowest terms. *Ans.* $\frac{4}{11}$.
6. Reduce $\frac{825}{908}$ to its lowest terms. *Ans.* $\frac{53}{58}$.
7. Reduce $\frac{53}{84}$ to its lowest terms. *Ans.* $\frac{2}{13}$.
8. Reduce $\frac{224}{1484}$ to its lowest terms. *Ans.* $\frac{7}{221}$.
9. Reduce $\frac{3746}{8888}$ to its lowest terms. *Ans.* $\frac{1873}{4444}$.
10. Reduce $\frac{5184}{912}$ to its lowest terms. *Ans.* $\frac{3}{2}$.
11. Reduce $\frac{1344}{1344}$ to its lowest terms. *Ans.* $\frac{7}{8}$.
12. Reduce $\frac{5328}{5328}$ to its lowest terms. *Ans.* $\frac{11}{11}$.

which remains in the second place is 10; but 2 measures 10; therefore the whole is divisible by 2.

2. If any number terminates on the right hand with a cypher, or 5, the whole is divisible by 5; for every unit which remains in the second place is 10; but 5 measures every multiple of 10; therefore the whole is divisible by 5.

3. If the two right-hand figures of any number are divisible by 4, the whole is divisible by 4; for every unit which remains in the third place is 100; but 4 measures every multiple of 100; therefore the whole is divisible by 4.

4. If the three right-hand figures of any number are divisible by 8, the whole is divisible by 8; for every unit which remains in the fourth place is 1000; but 8 measures every multiple of 1000; therefore the whole is divisible by 8.

5. If the sum of the digits constituting any number be divisible by 3 or 9, the whole is divisible by 3 or 9. This theorem has already been demonstrated in the notes on addition.

6. If the sum of the digits constituting any number be divisible by 6, and the right-hand digit by 2, the whole is divisible by 6; for by the data it is divisible both by 2 and 3.

7. If the sum of the 1st, 3d, 5th, &c. digits constituting any number be equal to that of the 2d, 4th, 6th, &c. that number is divisible by 11; for if a, b, c, d, e, m, n , be the digits, constituting any number, its digits, when multiplied by 11, will become

(8)	(7)	(6)	(5)	(4)	(3)	(2)	(1)
a	$a+b$	$b+c$	$c+d$	$d+e$	$e+m$	$m+n$	n

where the odd terms are = to the even.

PROBLEM II.

Any mixed number being given, to reduce it to its equivalent improper fraction.

RULE *.

Multiply the whole number by the denominator of the fraction, and add the numerator to the product; then that sum written above the denominator will form the fraction required.

EXAMPLES.

1. Reduce $64\frac{5}{8}$ to an improper fraction. *Ans.* $517\frac{5}{8}$.
2. Reduce $144\frac{3}{7}$ to an improper fraction. *Ans.* $1011\frac{3}{7}$.
3. Reduce $227\frac{4}{11}$ to an improper fraction. *Ans.* $2501\frac{4}{11}$.
4. Reduce $496\frac{1\frac{1}{2}}{1\frac{1}{2}}$ to an improper fraction. *Ans.* $596\frac{3}{2}$.
5. Reduce $692\frac{4}{5}$ to an improper fraction. *Ans.* $8900\frac{4}{5}$.
6. Reduce $872\frac{3}{4}$ to an improper fraction. *Ans.* $3491\frac{3}{4}$.
7. Reduce $794\frac{5}{9}$ to an improper fraction. *Ans.* $6278\frac{5}{9}$.
8. Reduce $989\frac{6}{7}$ to an improper fraction. *Ans.* $7423\frac{6}{7}$.
9. Reduce $900\frac{7}{8}$ to an improper fraction. *Ans.* $7297\frac{7}{8}$.
10. Reduce $886\frac{1}{2}$ to an improper fraction. *Ans.* $2017\frac{1}{2}$.
11. Reduce $1089\frac{2}{3}$ to an improper fraction. *Ans.* $6645\frac{2}{3}$.
12. Reduce $2867\frac{1}{4}$ to an improper fraction. *Ans.* $40151\frac{1}{4}$.

PROBLEM III.

Any improper fraction being given, to reduce it to its equivalent whole or mixed number.

RULE *.

Divide the numerator by the denominator, and the quotient will be the whole number; the remainder, if any, will be the numerator, and the divisor the denominator of a fraction, which added to the quotient will form the mixed number required.

* The whole number is here multiplied and divided by the same quantity; and therefore its value remains unchanged.

† This rule being only the reverse of the former, needs no farther explanation.

EXAM

EXAMPLES.

1. Reduce $5\frac{1}{8}$ to its equivalent whole or mixed number.
Ans. $64\frac{3}{8}$.
2. Reduce $10\frac{1}{7}$ to its equivalent whole or mixed number.
Ans. $144\frac{3}{7}$.
3. Reduce $2\frac{5}{11}$ to its equivalent whole or mixed number.
Ans. $227\frac{4}{11}$.
4. Reduce $5\frac{9}{12}$ to its equivalent whole or mixed number.
Ans. $496\frac{1}{2}$.
5. Reduce $9\frac{0}{3}$ to its equivalent whole or mixed number.
Ans. $692\frac{4}{3}$.
6. Reduce $3\frac{4}{9}$ to its equivalent whole or mixed number.
Ans. $872\frac{2}{9}$.
7. Reduce $6\frac{2}{7}$ to its equivalent whole or mixed number.
Ans. $794\frac{5}{9}$.
8. Reduce $7\frac{4}{7}$ to its equivalent whole or mixed number.
Ans. $989\frac{6}{3}$.
9. Reduce $7\frac{2}{11}$ to its equivalent whole or mixed number.
Ans. $900\frac{7}{1}$.
10. Reduce $2\frac{3}{2}$ to its equivalent whole or mixed number.
Ans. $886\frac{1}{2}$.
11. Reduce $6\frac{4}{8}$ to its equivalent whole or mixed number.
Ans. $1089\frac{3}{1}$.
12. Reduce $4\frac{0}{4}$ to its equivalent whole or mixed number.
Ans. $2867\frac{1}{1}$.

PROBLEM IV.

Any compounded fraction being given, to reduce it to an equivalent simple one.

RULE *.

Multiply all the numerators together for the numerator, and all the denominators together for the denominator of the simple fraction required.

* The reason of this rule will appear from the following example: let $\frac{a}{b}$ th part of $\frac{c}{d}$ be required. Then $\frac{c}{d}$ may be considered as an unit, and therefore $b : a :: \frac{c}{d} : \frac{ac}{bd} = \frac{a}{b} \times \frac{c}{d}$; and universally however many ratios are given, that required is compounded of the whole.

EXAMPLES.

1. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ to a simple fraction. *Ans.* $\frac{1}{3}$.
2. Reduce $\frac{3}{4}$ of $\frac{5}{8}$ to a simple fraction. *Ans.* $\frac{15}{32}$.
3. Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$ to a simple fraction. *Ans.* $\frac{30}{120} = \frac{1}{4}$.
4. Reduce $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{4}{11}$ to a simple fraction. *Ans.* $\frac{24}{132} = \frac{2}{11}$.
5. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{8}{11}$ to a simple fraction. *Ans.* $\frac{48}{132} = \frac{4}{11}$.
6. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{4}{5}$ of 5 to a simple fraction. *Ans.* $\frac{20}{30} = \frac{2}{3}$.
7. Reduce $\frac{3}{7}$ of $\frac{5}{8}$ of $\frac{7}{2}$ to a simple fraction. *Ans.* $\frac{70}{112} = \frac{5}{8}$.
8. Reduce $\frac{2}{5}$ of $\frac{5}{8}$ of $3\frac{1}{2}$ to a simple fraction. *Ans.* $\frac{70}{80} = \frac{7}{8}$.
9. Reduce $\frac{4}{5}$ of $\frac{3}{7}$ of $1\frac{1}{2}$ to a simple fraction. *Ans.* $\frac{36}{175} = \frac{18}{875}$.
10. Reduce $\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{1}{2}$ to a simple fraction. *Ans.* $\frac{15}{224} = \frac{15}{224}$.
11. Reduce $\frac{4}{11}$ of $\frac{5}{12}$ of $2\frac{1}{2}$ to a simple fraction. *Ans.* $\frac{60}{264} = \frac{5}{22}$.
12. Reduce $\frac{5}{16}$ of $\frac{8}{15}$ of $\frac{1}{2}$ to a simple fraction. *Ans.* $\frac{40}{240} = \frac{1}{6}$.

PROBLEM V.

Any multitude of fractions with different denominators being given, to reduce them to equivalent fractions, having a common denominator.

RULE *.

1. Multiply each numerator into all the denominators, except its own, for a new numerator; and all the denominators continually for a common denominator.
2. But if the common denominator be required a minimum, the least common multiple of all the given denominators will be that required; then, as each denominator is to its numerator, so is the common denominator to the several new numerators.

* The truth of this rule will appear thus: Let $\frac{a}{b}, \frac{c}{d}, \frac{e}{r}, \frac{n}{m}$ be any given fractions to be reduced to their equivalents with a common denominator; then the fractions ordered as per rule, will appear thus: $\frac{a \times d \times r \times m}{b \times d \times r \times m}, \frac{c \times b \times r \times m}{d \times b \times r \times m}, \frac{e \times b \times d \times m}{r \times b \times d \times m}, \frac{n \times b \times d \times r}{m \times b \times d \times r}$; which are evidently equal in value to the former, both terms being multiplied and divided by the same quantities.

EXAM^{PS}

REDUCTION OF VULGAR FRACTIONS.

73

EXAMPLES.

Reduce	to a common denominator	Answers.
1. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$	$\frac{24}{48}, \frac{32}{48}, \frac{30}{48}$	$= \frac{12}{24}, \frac{16}{24}, \frac{15}{24}$
2. $\frac{1}{2}, \frac{1}{3}, \frac{2}{4}, \frac{5}{6}$	$\frac{72}{144}, \frac{48}{144}, \frac{120}{144}, \frac{120}{144}$	$= \frac{6}{12}, \frac{4}{12}, \frac{10}{12}, \frac{10}{12}$
3. $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{7}{10}$	$\frac{60}{180}, \frac{125}{180}, \frac{144}{180}, \frac{144}{180}$	$= \frac{20}{60}, \frac{45}{60}, \frac{48}{60}, \frac{48}{60}$
4. $\frac{2}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{10}$	$\frac{540}{900}, \frac{900}{900}, \frac{1350}{900}, \frac{945}{900}$	$= \frac{36}{60}, \frac{60}{60}, \frac{90}{60}, \frac{63}{60}$
5. $\frac{1}{4}, \frac{2}{3}, \frac{4}{5}, \frac{3}{7}$	$\frac{1500}{6000}, \frac{3600}{6000}, \frac{1600}{6000}, \frac{900}{6000}$	$= \frac{15}{60}, \frac{26}{60}, \frac{16}{60}, \frac{9}{60}$
6. $\frac{1}{2}, \frac{2}{3}, \frac{5}{7}, \frac{3}{10}$	$\frac{3072}{3080}, \frac{2256}{3080}, \frac{7680}{3080}, \frac{6912}{3080}$	$= \frac{4}{8}, \frac{42}{48}, \frac{19}{48}, \frac{28}{48}$
7. $\frac{2}{3}, \frac{5}{6}, \frac{4}{7}, \frac{5}{12}$	$\frac{1208}{1944}, \frac{1620}{1944}, \frac{364}{1944}, \frac{810}{1944}$	$= \frac{24}{36}, \frac{30}{36}, \frac{16}{36}, \frac{15}{36}$
8. $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{1}{4}$	$\frac{10780}{23716}, \frac{20318}{23716}, \frac{3234}{23716}, \frac{1594}{23716}$	$= \frac{70}{134}, \frac{132}{134}, \frac{21}{134}, \frac{11}{134}$
9. $\frac{1}{6}, \frac{3}{8}, \frac{5}{10}, \frac{1}{12}$	$\frac{960}{5760}, \frac{2160}{5760}, \frac{2880}{5760}, \frac{3280}{5760}$	$= \frac{4}{24}, \frac{9}{24}, \frac{12}{24}, \frac{23}{24}$
10. $\frac{3}{8}, \frac{1}{5}, \frac{5}{12}, \frac{7}{10}$	$\frac{3456}{9216}, \frac{1836}{9216}, \frac{3840}{9216}, \frac{4032}{9216}$	$= \frac{18}{48}, \frac{8}{48}, \frac{20}{48}, \frac{21}{48}$
11. $\frac{7}{8}, \frac{1}{2}, \frac{3}{10}, \frac{2}{3}$	$\frac{32256}{3080}, \frac{15360}{3080}, \frac{6912}{3080}, \frac{16896}{3080}$	$= \frac{47}{78}, \frac{20}{78}, \frac{9}{78}, \frac{23}{78}$
12. $\frac{1}{2}$ of $\frac{1}{3}, \frac{2}{3}$ of $\frac{1}{4}, \frac{3}{4}$ of 5.	$\frac{40}{128}, \frac{64}{128}, \frac{160}{128}$	$= \frac{5}{16}, \frac{8}{16}, \frac{20}{16}$

PROBLEM VI.

Any number, either whole or fracted, being given, to reduce it to an equivalent fraction, one of the terms of which shall be any assigned number.

RULE*.

Find a fourth proportional of the two terms of the whole number or fraction, and the number assigned; the fourth pro-

* This is evident from the rule of proportion, without any further illustration.

H

portional

portional thus found, will be the other term of the fraction required.

N. B. Numbers, whether whole or fracted, consist of two terms; all whole numbers having unity for their common consequent.

EXAMPLES.

1. Reduce 22	} to an equivalent fraction, the	Denominator of which shall be 12.	<i>Ans.</i> $\frac{264}{12}$
2. Reduce $\frac{5}{12}$		Numerator of which shall be 25.	<i>Ans.</i> $\frac{25}{86}$
3. Reduce $\frac{7}{11}$		Denominator of which shall be 14.	<i>Ans.</i> $\frac{810}{14}$
4. Reduce $\frac{14}{25}$		Numerator of which shall be 5.	<i>Ans.</i> $\frac{5}{813}$
5. Reduce $\frac{88}{133}$		Denominator of which shall be 55.	<i>Ans.</i> $\frac{3652}{133}$
6. Reduce $\frac{88}{193}$		Numerator of which shall be $6\frac{3}{5}$.	<i>Ans.</i> $\frac{63}{145}$
7. Reduce 24		Denominator of which shall be 5.	<i>Ans.</i> $\frac{120}{5}$
8. Reduce 48		Numerator of which shall be 16.	<i>Ans.</i> $\frac{16}{13}$
9. Reduce $\frac{2}{3}$		Denominator of which shall be 18.	<i>Ans.</i> $\frac{12}{18}$
10. Reduce $\frac{3}{4}$		Numerator of which shall be 12.	<i>Ans.</i> $\frac{12}{10}$
11. Reduce $\frac{5}{13}$		Denominator of which shall be 28.	<i>Ans.</i> $\frac{1010}{28}$
12. Reduce $\frac{36}{83}$		Numerator of which shall be 4.	<i>Ans.</i> $\frac{4}{93}$

PROBLEM VII.

Any complex fraction being given, to reduce it to an equivalent simple expression.

RULE*.

Reduce the mixed terms to improper fractions, and multiply both terms of the fraction by their denominators; the resulting products will form the terms of the simple fraction required.

EXAMPLES.

1. Reduce $\frac{44\frac{3}{4}}{58}$ to an equivalent simple fraction.

$$\text{Ans. } \frac{174}{174} = \frac{67}{67}.$$

2. Reduce $\frac{34\frac{4}{9}}{54}$ to an equivalent simple fraction.

$$\text{Ans. } \frac{310}{243} = \frac{155}{121\frac{1}{2}}.$$

3. Reduce $\frac{6}{54\frac{3}{4}}$ to an equivalent simple fraction.

$$\text{Ans. } \frac{66}{198} = \frac{11}{33}.$$

4. Reduce $\frac{36}{99\frac{3}{7}}$ to an equivalent simple fraction.

$$\text{Ans. } \frac{252}{891} = \frac{2}{7}.$$

5. Reduce $\frac{25}{64\frac{1}{6}}$ to an equivalent simple fraction.

$$\text{Ans. } \frac{150}{384} = \frac{25}{64}.$$

6. Reduce $\frac{8\frac{10}{11}}{14}$ to an equivalent simple fraction.

$$\text{Ans. } \frac{98}{154} = \frac{7}{11}.$$

7. Reduce $\frac{5}{8\frac{1}{4}}$ to an equivalent simple fraction.

$$\text{Ans. } \frac{70}{112} = \frac{5}{8}.$$

8. Reduce $\frac{36\frac{5}{13}}{55}$ to an equivalent simple fraction.

$$\text{Ans. } \frac{4840}{7513} = \frac{88}{133}.$$

* It is plain that both terms are here multiplied by the same quantities; and therefore the ratio remains the same.

9. Reduce $\frac{10\frac{10}{13}}{28}$ to an equivalent simple fraction.

$$\text{Ans. } \frac{140}{364} = \frac{5}{13}.$$

10. Reduce $\frac{6\frac{3}{5}}{14\frac{5}{8}}$ to an equivalent simple fraction.

$$\text{Ans. } \frac{264}{385} = \frac{24}{35}.$$

11. Reduce $\frac{4\frac{2}{3}}{12\frac{2}{3}}$ to an equivalent simple fraction.

$$\text{Ans. } \frac{70}{148} = \frac{35}{74}.$$

12. Reduce $\frac{23\frac{3}{4}}{67\frac{1}{2}}$ to an equivalent simple fraction.

$$\text{Ans. } \frac{180}{340} = \frac{9}{17}.$$

PROBLEM VIII.

The ratio of two integers and any fraction of the one being given, to reduce that fraction to an equivalent expression of the other.

RULE *.

As the integer of the required expression is to that of the given fraction, so is the fraction given to that required: or, the fractions are reciprocally as their integers.

EXAMPLES.

1. Reduce $\frac{2}{3}$ of a shilling to the fraction of a pound.

$$\text{Ans. } \frac{2}{360} = \frac{1}{180}.$$

2. Reduce $\frac{5}{8}$ of a penny to the fraction of a pound.

$$\text{Ans. } \frac{5}{480} = \frac{1}{96}.$$

3. Reduce $\frac{1}{3}$ of a pound to the fraction of a penny.

$$\text{Ans. } \frac{240}{3} = 80.$$

* That the fractions should be reciprocally as the integers, appears evident thus:

Let the ratio of the given integers be $a : b = \frac{a}{b}$, and the fraction of $a = \frac{c}{d}$; therefore, per prop. 4th, $\frac{a}{b} \times \frac{c}{d} =$ the fraction required; (i. e.) $b : a :: \frac{c}{d} : \frac{ac}{bd}$. Let (e. g.) $a = 20$, and $b = 1$; $c = 3$, and $d = 4$; then $\frac{20}{1} \times \frac{3}{4} = \frac{60}{4} = 15$ or $\frac{3}{4}$ of a pound expressed in shillings.

4. Reduce

4. Reduce $\frac{5}{10}$ of a crown to the fraction of a guinea.
Ans. $\frac{25}{210} = \frac{5}{42}$.
5. Reduce $\frac{2}{3}$ of a grain troy to the fraction of a pound.
Ans. $\frac{17280}{17280} = \frac{1}{840}$.
6. Reduce $\frac{1}{80}$ of a pound troy to the fraction of a grain.
Ans. $\frac{5760}{480} = \frac{12}{1}$.
7. Reduce $\frac{5}{8}$ of a pound avoird. to the fraction of a ton.
Ans. $\frac{17920}{17920} = \frac{1}{336}$.
8. Reduce $\frac{1}{120}$ of a ton to the fraction of a pound avoird.
Ans. $\frac{2240}{1120} = \frac{2}{1}$.
9. Reduce $\frac{2}{3}$ of a pint of wine to the fraction of a tun.
Ans. $\frac{10680}{10680} = \frac{1}{3360}$.
10. Reduce $\frac{3}{728}$ of a butt of beer to the fraction of a pint.
Ans. $\frac{2592}{840} = \frac{3}{10}$.
11. Reduce $\frac{5}{8}$ of a yard to the fraction of a mile.
Ans. $\frac{5040}{17920} = \frac{1}{2816}$.
12. Reduce $\frac{5}{7}$ of a minute to the fraction of a month.
Ans. $\frac{5224}{18480} = \frac{1}{3548}$.

PROBLEM IX.

Any number compounded of different denominations being given, to reduce it to an equivalent fraction of a given integer.

RULE *.

Reduce the compounded number to one denomination, and find an equivalent fraction of the given integer, by the rule for the solution of problem the eighth.

EXAMPLES.

1. Reduce 13s. 4d. to the fraction of a pound sterling.
Ans. $\frac{160}{240} = \frac{2}{3}$.
2. Reduce 2s. 7½d. to the fraction of a guinea.
Ans. $\frac{63}{84} = \frac{3}{4}$.
3. Reduce 19s. 8d. 2¾q. to the fraction of a moidore.
Ans. $\frac{6624}{9728} = \frac{6}{7}$.
4. Reduce 6 oz. 12 dwts. 12 gr. to the fraction of a pound troy.
Ans. $\frac{3180}{5760} = \frac{53}{96}$.
5. Reduce 4c. 2q. 14lb. to the fraction of a ton.
Ans. $\frac{518}{2240} = \frac{37}{160}$.

* This is plain per problem 8th.

6. Reduce 4 *fur.* 20 *pol.* to the fraction of a mile.
Ans. $\frac{180}{16} = \frac{9}{8}$.
7. Reduce 2 *r.* 20 *pol.* 15 *yd.* to the fraction of an acre.
Ans. $\frac{2040}{121} = \frac{76}{121}$.
8. Reduce 3 *bd.* 31 *gal.* 2 *qrt.* of wine to the fraction of a tun,
Ans. $\frac{882}{1008} = \frac{7}{8}$.
9. Reduce 6 *gal.* 3 $\frac{1}{2}$ *qrt.* of beer to the fraction of a butt.
Ans. $\frac{128}{2180} = \frac{23}{380}$.
10. Reduce 4 *buf.* 1 *pec.* 1 *gal.* 2 $\frac{2}{3}$ *qr.* of corn to the fraction of a quarter.
Ans. $\frac{1280}{2384} = \frac{5}{9}$.
11. Reduce 8 *b.* 26 *m.* 40 *f.* of time to the fraction of a day.
Ans. $\frac{30400}{86400} = \frac{19}{54}$.
12. Reduce 2 *w.* 3 *d.* 12 *b.* to the fraction of a month.
Ans. $\frac{420}{56} = \frac{35}{4}$.

PROBLEM X.

The fraction of any integer being given, to determine its value in the known parts of that integer.

RULE*.

1. Multiply the integer, if greater than unity, or the number expressing its value in the next lower order, if unity, by the numerator of the fraction, and divide by the denominator.
2. Multiply the remainder, or numerator of the new fraction, by the value of its integer in the next lower order, and divide by the denominator as before, continuing the process to the lowest denomination, and the several quotients will be the value required.

EXAMPLES.

1. Reduce $\frac{2}{3}$ of a pound sterling to the equivalent parts of the integer.
Ans. 13s. 4d.
2. Reduce $\frac{1}{4}$ of a guinea to the equivalent parts of the integer.
Ans. 2s. 7d. 2q.
3. Reduce $\frac{6}{7}$ of a moidore to the equivalent parts of the integer.
Ans. 19s. 8d. 2 $\frac{2}{3}$ q.
4. Reduce $\frac{3}{8}$ of a pound troy to the equivalent parts of the integer.
Ans. 6oz. 12dwts. 12gr.
5. Reduce $\frac{37}{80}$ of a ton to the equivalent parts of the integer.
Ans. 4c. 2q. 14lb.

* This is only the reverse of problem 9th.

ADDITION OF VULGAR FRACTIONS.

79

6. Reduce $\frac{1}{2}$ of a mile to the equivalent parts of the integer;
Ans. 4 fur. 20 pol.
7. Reduce $\frac{7}{12}$ of an acre to the equivalent parts of the integer.
Ans. 2 r. 20 p. 15 yd.
8. Reduce $\frac{1}{4}$ of a tun of wine to the equivalent parts of the integer.
Ans. 3 bhd. 31 gal. 2 qt.
9. Reduce $\frac{2}{3}$ of a butt of beer to the equivalent parts of the integer.
Ans. 6 gal. 3 qt.
10. Reduce $\frac{1}{4}$ of a quarter of corn to the equivalent parts of the integer.
Ans. 4 b. 1 p. 1 g. 2 qt.
11. Reduce $\frac{1}{4}$ of a day to the equivalent parts of the integer.
Ans. 8 h. 26 m. 40 s.
12. Reduce $\frac{2}{3}$ of 3 w. 4 d. to the equivalent parts of the integer.
Ans. 2 w. 2 d. 16 h.

ADDITION OF VULGAR FRACTIONS.

ADDITION OF VULGAR FRACTIONS is the mode of collecting into one sum the several fracted parts of homogeneous integers; by performing which the following problem is solved.

P R O B L E M X I.

Several fracted parts of homogeneous integers being given, to determine their sum.

R U L E *.

1. Reduce compound and complex fractions to simple ones; mixed numbers to improper fractions; and all to the same integer and denominator if different.
2. Add all the numerators together, and place their sum over the common denominator, for the required sum of the given fractions.

E X A M P L E S.

1. What is the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$? *Ans.* $\frac{106}{96} = 2\frac{1}{24}$.
2. What is the sum of $\frac{5}{8}$, $7\frac{1}{2}$, and $\frac{1}{3}$ of $\frac{3}{4}$? *Ans.* $\frac{536}{84} = 8\frac{1}{21}$.

* Neither sum nor difference of magnitudes can be assigned until some common standard be assumed, to which the measures of all must be referred: but these measures must also be expressed on the same scale: hence the necessity for reducing the given fractions to the same integer and denominator, if different.

3. What

SUBTRACTION OF VULGAR FRACTIONS.

3. What is the sum of $\frac{2}{3}l.$, $\frac{3}{4}s.$ and $\frac{1}{2}d.$ *Ans.* 14s. 1d. 2q.
4. What is the sum of $\frac{3}{8}m.$, $\frac{4}{5}f.$, $\frac{1}{2}$ pole, and $\frac{2}{3}$ of a foot? *Ans.* 3f. 32p. 2yd. 2ft. 11in.
5. What is the sum of $\frac{1}{3}l.$, $\frac{3}{4}$ guinea, and $\frac{1}{2}$ of a moidore? *Ans.* 1l. 13s. 11d.
6. What is the sum of $\frac{1}{2}lb.$, $\frac{3}{4}oz.$ and $\frac{5}{8}$ of a pennyweight? *Ans.* 6oz. 15dwt. 20gr.
7. What is the sum of $\frac{5}{8}t.$, $\frac{3}{4}cwt.$, $\frac{1}{8}q.$ and $\frac{1}{2}$ of an ounce? *Ans.* 13cwt. 1q. 3lb. 8oz. 8dr.
8. What is the sum of $\frac{5}{8}ac.$, $\frac{4}{5}r.$, $\frac{4}{11}p.$ and $\frac{1}{2}$ of a yard? *Ans.* 3r. 12p. 11yd. 1f. 6in.
9. What is the sum of $\frac{3}{4}tun.$, $\frac{7}{8}bhd.$ and $\frac{5}{8}$ of a gallon of wine? *Ans.* 3bhd. 49gal. 5 pints.
10. What is the sum of $\frac{3}{10}mon.$, $\frac{5}{8}w.$ and $\frac{1}{12}$ of a day? *Ans.* 1w. 6d. 8h. 36min.
11. What is the sum of $\frac{2\frac{1}{2}}{3\frac{1}{3}}l.$ and $\frac{5\frac{1}{4}}{7}$ of a shilling? *Ans.* 15s. 9d.
12. What is the sum of $\frac{2}{3}$ of $\frac{1}{4}$ of 1500l. and $\frac{3}{4}$ of 5l. 12s.? *Ans.* 629l. 4s.

SUBTRACTION OF VULGAR FRACTIONS.

SUBTRACTION OF VULGAR FRACTIONS is the mode of determining the difference of the fracted parts of homogeneous integers: by determining which the following problem is solved.

PROBLEM XII.

Any two fracted parts of homogeneous integers being given, to determine their difference.

RULE.

Prepare the given numbers as in Addition, and place the difference of the numerators above the common denominator; the expression thus formed will be the required difference.

EXAMPLES.

1. What is the difference of $\frac{3}{4}$ and $\frac{5}{8}$? *Ans.* $\frac{2^4 - 5^0}{3^4 \cdot 2^0} = \frac{4}{3^4 \cdot 2} = \frac{1}{3}$.
2. What is the difference of $\frac{7}{8}$ and $\frac{3}{4}$ of $\frac{2}{3}$? *Ans.* $\frac{3^4 \cdot 2^4 - 3^4 \cdot 2^0}{4^4 \cdot 3^0} = \frac{11}{4^4 \cdot 3}$.
3. What

MULTIPLICATION OF VULGAR FRACTIONS. 81

3. What is the difference of $12\frac{5}{8}$ and $8\frac{3}{5}$? *Ans.* $7\frac{7}{40} = 4\frac{7}{10}$
4. What is the difference of $6\frac{1}{2}$ and $\frac{5\frac{1}{3}}{12\frac{2}{3}}$? *Ans.* $2\frac{1}{2} = \frac{5}{2}$
5. What is the difference of $\frac{2}{3}l.$ and $\frac{3}{4}s.$ *Ans.* $12s. 11d. 2q.$
6. What is the difference of $\frac{1}{3}l.$ and $\frac{5}{11}$ of a crown? *Ans.* $16s. 2d. 1\frac{2}{11}q.$
7. What is the difference of $\frac{3}{11}$ of $23l. 3s. 10d.$ and $\frac{5}{6}$ of a moidore? *Ans.* $5l. 7s. 4d.$
8. What is the difference of $7lb.$ troy and $\frac{3}{5}$ of an ounce? *Ans.* $90z. 18dwt.$
9. What is the difference of $\frac{3}{8}$ ton and $\frac{5}{12}$ of cwt. *Ans.* $7c. 0q. 9lb. 5oz. 5\frac{1}{3}dr.$
10. What is the difference of $\frac{5}{6}$ league and $\frac{1}{12}$ of a mile? *Ans.* $1m. 4fur. 26p. 3yd. 2feet.$
11. What is the difference of $\frac{5}{12}$ tun and $\frac{1}{3}$ of a bhd.? *Ans.* $1bhd. 14gall.$
12. What is the difference of $\frac{1}{12}$ year and $\frac{1}{4}$ of $\frac{1}{4}$ of a month? *Ans.* $11m. 3w. 1d. 18hours.$

MULTIPLICATION OF VULGAR FRACTIONS.

MULTIPLICATION OF VULGAR FRACTIONS is the mode of determining the product of several fracted parts of given integers; by performing which the following problem is solved.

P R O B L E M XIII.

Any number of fractional expressions being given, to determine their continued product.

R U L E *.

1. Reduce mixed numbers to improper fractions, and complex fractions to simple ones.
2. Multi-

* The product of any number of fractions is expressed by a ratio compounded of the whole, and is the same with a compounded fraction; the continual product of all the consequents expresses the number of parts into which the unit is supposed to be divided, and that of all the antecedents expresses the number of these parts to be taken.

2. Multiply all the numerators together for the numerator, and all the denominators together for the denominator of the product required.

EXAMPLES.

1. What is the product of $\frac{2}{7}$, $\frac{5}{8}$, and $\frac{7}{12}$? *Ans.* $\frac{7^0}{8^7 2} = \frac{5}{48}$.
2. What is the product of $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{6}{11}$? *Ans.* $\frac{7^2}{8^9 3} = \frac{3^4}{23^4}$.
3. What is the product of $\frac{5}{8}$, $\frac{3}{5}$, and $\frac{2}{3}$ of $\frac{1}{2}$? *Ans.* $\frac{3^0}{1^8 0} = \frac{1}{8}$.
4. What is the product of $\frac{1^5}{1^8}$, $\frac{9}{1^3}$, and $\frac{3}{4}$ of $\frac{5}{4}$? *Ans.* $\frac{1^8 0^0}{7^8 8^0} = \frac{1^5}{8^4}$.
5. What is the product of $\frac{2^1}{3^3}$, $\frac{2^2}{4^5}$, and $\frac{9}{1^1}$ of $4^{\frac{1}{2}}$? *Ans.* $\frac{3^7 4^2 2}{2^8 7^0} = 1^{\frac{9}{2}}$.
6. What is the product of $\frac{4}{1^1}$, $\frac{8}{3^3}$, and $\frac{4^4}{3^2}$ of $2^{\frac{3}{8}}$? *Ans.* $\frac{2^6 7^5 2}{8^2 8} = \frac{1^0}{8^2}$.
7. What is the product of $\frac{2}{9}$ of $\frac{2}{5}$, 9, and $\frac{5}{8}$ of $6^{\frac{4}{7}}$? *Ans.* $\frac{8^2 8^0}{2^3 2^0} = 3^2$.
8. What is the product of $3^{\frac{2}{7}}$, $\frac{1}{2}$, and $4^{\frac{1}{3}}$? *Ans.* $\frac{3^3 5^8}{4^0 2^2} = 7^{\frac{6^2}{21}}$.
9. What is the product of $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{8}$, and $12^{\frac{2}{3}}$? *Ans.* $\frac{1^0 2^0}{8^0 0^0} = 3^{\frac{1}{2}}$.
10. What is the product of $\frac{1^3}{1^3}$, $\frac{2^5}{2^6}$, $\frac{1^2}{3^3}$, and $8^{\frac{3}{4}}$? *Ans.* $\frac{1^3 6^5 0^0}{1^4 8^0 8^0} = 2^{\frac{1}{2}}$.
11. What is the product of $\frac{1^2 1}{2^4 8}$, $\frac{4}{1^1}$, $\frac{6}{3^3}$, and $\frac{4^2}{1^1}$? *Ans.* $\frac{4^2}{4^8 7^8 7^2 8} = \frac{5}{3^3 8}$.
12. What is the product of $\frac{3^1}{2}$, $\frac{8}{3^1}$, $\frac{1^1}{2^1}$, and $10^{\frac{1}{1}}$? *Ans.* $\frac{3^0 5^5 1^6}{3^0 5^5 3^6} = 1$.

DIVISION OF VULGAR FRACTIONS.

DIVISION OF VULGAR FRACTIONS is the mode of finding a whole or fracted number, which will be to unity as the dividend to the divisor; by performing which the following problem is solved.

In integers, unity is the common consequent to all; therefore if the several numerators of the given fractions were supposed integers, the value of the integer product would be to that of the fracted product, as the denominator of the fracted product to unity.

PRO.

PROBLEM XIV.

Any two fractional expressions being given, to determine the ratio of the one to the other, if of the same kind, or a fourth proportional to the two given fractions and unity.

RULE*.

Prepare the terms as in Multiplication, and multiply the dividend by the reciprocal of the divisor (or divisor inverted); the product will be that required.

EXAMPLES.

1. What is the quotient of $\frac{1}{2}\frac{6}{5}$ by $\frac{4}{3}$? *Ans.* $\frac{9}{10} = \frac{9}{10}$.
2. What is the quotient of $\frac{7}{1}\frac{3}{5}$ by $\frac{3}{5}$? *Ans.* $\frac{3}{4}\frac{5}{5} = \frac{7}{4}$.
3. What is the quotient of $\frac{2}{3}\frac{1}{2}$ by $\frac{3}{4}$? *Ans.* $\frac{8}{9}\frac{4}{4} = \frac{8}{9}$.
4. What is the quotient of $\frac{3}{6}\frac{4}{3}$ by $\frac{1}{2}\frac{7}{7}$? *Ans.* $\frac{5}{1}\frac{1}{8}\frac{3}{7} = \frac{6}{7}$.
5. What is the quotient of $\frac{1}{3}\frac{2}{5}$ by $\frac{3}{5}$? *Ans.* $\frac{6}{1}\frac{0}{3} = \frac{4}{7}$.
6. What is the quotient of $\frac{4}{8}\frac{5}{4}$ by $\frac{9}{8}$? *Ans.* $\frac{3}{3}\frac{6}{7}\frac{0}{8} = \frac{8}{8}$.
7. What is the quotient of $\frac{9}{1}\frac{0}{8}$ by 3? *Ans.* $\frac{9}{4}\frac{0}{8} = \frac{3}{10}$.
8. What is the quotient of $12\frac{3}{5}$ by $\frac{2}{3}\frac{1}{6}$? *Ans.* $\frac{1}{1}\frac{8}{0}\frac{9}{5} = 18$.
9. What is the quotient of $15\frac{5}{6}$ by $\frac{5}{6}$ of 7? *Ans.* $\frac{1}{3}\frac{2}{1}\frac{6}{5} = 4$.
10. What is the quotient of $7\frac{1}{3}$ by $9\frac{5}{8}$? *Ans.* $\frac{1}{2}\frac{5}{5}\frac{8}{8} = \frac{3}{4}\frac{3}{8}$.
11. What is the quotient of $\frac{4}{2}\frac{1}{2}$ by $\frac{8}{8}\frac{3}{7}\frac{1}{2}$? *Ans.* $\frac{8}{4}\frac{0}{5} = 2$.
12. What is the quotient of $2\frac{2}{3}$ by $\frac{1}{8}$ of $\frac{2}{11}$? *Ans.* $1\frac{4}{5}\frac{2}{2} = 242$.

* Division being the reverse of Multiplication, the rule will appear evident: for if it were required to divide $\frac{a}{b}$ by $\frac{c}{d}$, the quotient might be represented thus, $\frac{a}{b} \div \frac{c}{d}$; but if both terms be multiplied by $b d$,

will become $\frac{ad}{bc} = \frac{a}{b} \times \frac{d}{c}$.

SIMPLE

COMPOUND PROPORTION IN VULGAR FRACTIONS. 85

11. If a floor be $25\frac{1}{3}$ ells long, $9\frac{3}{4}$ ells wide, how many ells of matting $31\frac{1}{2}$ inches wide will cover the same?

Ans. $15\frac{3}{4}$ ells = 3164.

12. If I have $12\frac{3}{4}$ cwt. carried $36\frac{3}{4}$ miles for a given sum, how far may I have $6\frac{3}{4}$ tons carried for the same money?

Ans. $17\frac{1}{8}$ = $3\frac{1}{4}$ miles.

COMPOUND PROPORTION IN VULGAR FRACTIONS.

COMPOUND PROPORTION IN VULGAR FRACTIONS is the mode of finding a number, whole or fracted, the ratio of which to a given number of the same kind, shall be equal to that compounded of two, three, or more fractional expressions; by performing which the following problem is solved.

P R O B L E M XVI.

Any odd number of fractional expressions being given, which two and two are homogeneous, and their several connections known; to determine a number, whole or fracted, which shall with the odd term constitute a ratio equal to that compounded of the other terms.

R U L E.

1. Having made the necessary preparations for Multiplication, mark the terms, which from the data of the question, and the known principles of proportion, are found to be divisors.

2. Multiply continually the remaining terms and the reciprocals of the divisors; the resulting product will be that required.

E X A M P L E S.

1. If 50*l.* in 5 months gain $2\frac{3}{4}$ *l.* in what time will $13\frac{1}{2}$ *l.* gain $1\frac{1}{2}$ *l.*? *Ans.* $\frac{1}{3}$ year.

2. If $33\frac{1}{2}$ *l.* in $2\frac{2}{3}$ years gain $2\frac{2}{3}$ *l.* what sum will gain 16*l.* in $5\frac{1}{2}$ years? *Ans.* $2\frac{1}{2}$ = 85 $\frac{1}{2}$ *l.*

3. If I have $2\frac{3}{4}$ tons carried 100 $\frac{1}{4}$ miles for $5\frac{1}{2}$ *l.* how far may I have $44\frac{1}{2}$ tons carried for $25\frac{1}{2}$ pounds? *Ans.* $28\frac{3}{4}$ miles.

4. If a footman travels $180\frac{1}{2}$ miles in $6\frac{1}{8}$ days of $12\frac{1}{2}$ hours each, in how many days of $8\frac{1}{2}$ hours each may he travel 108 $\frac{1}{2}$ miles? *Ans.* $54\frac{9}{10}$ days.

5. If 12 men in $32\frac{1}{2}$ days ripe $16\frac{1}{4}$ acres, how many men will ripe $146\frac{1}{4}$ acres in $10\frac{5}{8}$ days of equal length with the former? *Ans.* 324 men.
6. If 16 men in $15\frac{1}{5}$ days build a wall $24\frac{1}{3}$ feet long, $6\frac{1}{2}$ feet high, and $2\frac{1}{4}$ feet thick; in how many days of equal length will 24 men build a wall $97\frac{1}{3}$ feet long, $19\frac{1}{2}$ feet high, and $4\frac{1}{2}$ feet thick? *Ans.* $243\frac{1}{5}$ days.

DECIMAL FRACTIONS.

INTEGERS and DECIMALS are so much alike in all their fundamental modes of operation, that the rules already given for the one include both; and nothing remains to be determined in these but the punctum separandi, or separating point, which will easily be discovered by the following rules and examples.

ADDITION OF DECIMALS.

RULE.

1. Arrange the numbers to be added as directed in addition of integers, (*i. e.*) place numbers of the same denominators directly under each other, by which the separating points of all will be in the same vertical line.

2. Add the numbers in every respect as integers, and place the separating point of the sum total in the said vertical line produced.

EXAMPLES.

783586.832586	5674252.3245678	67432486.75867
24573.45867	3256743.258065	3258673.2455
356742.5649678	4587532.658032	35674321.53428
9586.742583	74316.7456789	768432.258
24867.3258674	6758635.65432	6341285.54327
673251.256403	325873.486735	94286713.258
892164.5873	8743586.5683746	7324689.94565
2764772.7683772		

DECIMAL FRACTIONS.

87

5324567.5806325	8674301.23456	7456732.4586
56743.245867	7809256.789	3274.0035
247634.6745867	4325678.9132	674325.80023
3586422.0343248	56782.56789	6432564.325
7032586.743243	7437865.43258	325876.0558
673408.3206	248736.5032	7234567.89
5864325.4582347	4567890.035	1275486.4567
2345678.9023456	8767.3587	864325.89
7258732.5864325	9432586.74324	9458673.24586

SUBTRACTION OF DECIMALS.

R U L E.

1. Arrange the numbers as directed in Subtraction of integers, (i. e.) place the less number under the greater, the separating points in the same vertical line, and equal places equally remote.

2. Subtract the numbers in every respect as integers, and place the separating point of the difference in the said vertical line produced.

E X A M P L E S.

74586732.583245	69432456.789012	94586.74324567
32674368.427358	26583127.458673	46832.49673876
2386743.245674	8645867.4325	9673245.867897
58674.3245	4749658.3278678	386749.7867493

MULTIPLICATION OF DECIMALS.

R U L E *.

1. Arrange the factors from right to left, as if they were integers, and multiply in the same manner.

* The reason why the product should contain as many places of Decimals as there are in both factors, is thus manifest. Let $\frac{a}{10^m}$ be the two factors; their product $\frac{a}{10^m} \times \frac{b}{10^n} = \frac{ab}{10^{m+n}}$, where the number of places is equal to that in both factors.

2. Point off, from right to left, as many figures of the product as there are decimal places in both factors; and if the product is deficient, prefix cyphers to the left, to supply that deficiency.

EXAMPLES.

$$\begin{array}{r} 6743249.6324 \\ 734.765 \\ \hline \end{array}$$

$$\begin{array}{r} 337162481620 \\ 404594977944. \\ 472027474268.. \\ 269729985296... \\ 202297488972.... \\ 472027474268..... \\ \hline \end{array}$$

$$4954703806.1503860$$

$$\begin{array}{r} 7325864.3258674 \\ 854.326 \\ \hline \end{array}$$

$$\begin{array}{r} 439551859552044 \\ 146517286517348. \\ 219775929776022.. \\ 293034573034696... \\ 366293216293370.... \\ 586069146069392..... \\ \hline \end{array}$$

$$6258676366.0609923724$$

3. What is the product of 8216. 49735 and 30. 27 ?
Ans. 248713. 3747845.
4. What is the product of 16358. 7248 and 70. 4006 ?
Ans. 1151664. 04115488.
5. What is the product of 12345. 6789 and 1234. 56789 ?
Ans. 1524157. 8750190521.
6. What is the product of 92176. 00356 and 52. 0007091 ?
Ans. 4793217. 547124124396.

DIVISION OF DECIMALS.

RULE*.

1. Arrange the divisor and dividend as in integers, and divide in the same manner.

2. Point off, from right to left, as many figures of the quotient as the decimal places in the dividend exceed those in the divisor; and if the quotient is deficient, prefix cyphers to the left, to supply that deficiency.

* The reason of this rule is manifest: for since the dividend is equal to the product of the divisor and quotient, that ought to contain as many places as there are in both; therefore the number of decimal places in the quotient ought to be equal to the difference of places in the dividend and divisor.

EXAM-

EXAMPLES.

1. What is the quotient of 248713. 3747845 by 30. 27?
Ans. 8216. 49735.
2. What is the quotient of 1151664. 04115488 by 70. 4006?
Ans. 16358. 7248.
3. What is the quotient of 4954703806. 150386 by 734. 765?
Ans. 6743249. 6324.
4. What is the quotient of 6258676366. 0609923724 by 854. 326?
Ans. 7325864. 3258674.
5. What is the quotient of 1524157. 8750190521 by 1234. 56789?
Ans. 12345. 6789.
6. What is the quotient of 4793217. 547124124396 by 52. 0007091?
Ans. 92176. 00356.

REDUCTION OF DECIMALS.

REDUCTION OF DECIMALS is the mode of converting the known parts of given integers into equivalent Decimals; and, vice versa, the mode of converting given Decimals to the known parts of their integers, or to equivalent Decimals of different integers; by performing which the following problems are solved.

PROBLEM I.

Any vulgar fraction being given, to reduce it to an equivalent Decimal.

RULE*.

Annex cyphers at pleasure to the numerator, which divide by the denominator, and the quotient will be that required.

EXAM-

* If any two fractions are equivalent, their denominators will be directly as their numerators; but in this case both terms of the one are given, and the denominator of the other must be some power of 10, which may be carried on at pleasure by annexing cyphers; therefore the rule is manifest.

Among the various methods proposed for converting a vulgar fraction, the denominator of which is any prime number greater than what is common to divide by in one line, into a Decimal consisting of a great number of figures, that given by Mr. Colson, in page 162 of Sir Isaac Newton's Fluxions, is perhaps the best yet known, which I shall with others take the liberty to transcribe.

EXAMPLES.

1. Reduce $\frac{1}{2}$ to an equivalent decimal. *Ans.* .5.
2. Reduce $\frac{3}{4}$ to an equivalent decimal. *Ans.* .75.
3. Reduce $\frac{5}{8}$ to an equivalent decimal. *Ans.* .625
4. Reduce $\frac{7}{16}$ to an equivalent decimal. *Ans.* .4375.
5. Reduce $\frac{9}{32}$ to an equivalent decimal. *Ans.* .28125.
6. Reduce $\frac{11}{64}$ to an equivalent decimal. *Ans.* .171875.
7. Reduce $\frac{13}{128}$ to an equivalent decimal. *Ans.* .1015625.
8. Reduce $\frac{1}{3}$ to an equivalent decimal. *Ans.* .6.
9. Reduce $\frac{4}{5}$ to an equivalent decimal. *Ans.* .8.
10. Reduce $\frac{6}{125}$ to an equivalent decimal. *Ans.* .048.
11. Reduce $\frac{32}{25}$ to an equivalent decimal. *Ans.* .0512.
12. Reduce $\frac{128}{3125}$ to an equivalent decimal. *Ans.* .04096.
13. Reduce $\frac{768}{15625}$ to an equivalent decimal. *Ans.* .049152.
14. Reduce $\frac{6144}{78125}$ to an equivalent decimal. *Ans.* .0786432.
15. Reduce $\frac{264}{512}$ to an equivalent decimal. *Ans.* .56.
16. Reduce $\frac{598}{256}$ to an equivalent decimal. *Ans.* .224.

“Suppose (for instance) I would find the reciprocal of the prime number 29, or the value of the fraction $\frac{1}{29}$ in decimal numbers. I divide 10,000, &c. by 29, in the common way, so far as to find two or three of the first figures, or until the remainder becomes a single figure, and then I assume the supplement to complete the quotient. Thus I shall have $\frac{1}{29} = 0.03448 \frac{8}{29}$, for the complete quotient; which equation if I multiply by the numerator 8, it will give $\frac{8}{29} = 0.27584 \frac{64}{29}$ or rather $\frac{8}{29} = 0.27586 \frac{8}{29}$. I substitute this instead of the fraction in the first equation, and I shall have $\frac{1}{29} = 0.0344827586 \frac{6}{29}$. Again, I must multiply this equation by 6, and it will give $\frac{6}{29} = 0.2068965517 \frac{7}{29}$, and then by substitution $\frac{1}{29} = 0.03448275862068965517 \frac{7}{29}$. Again, I multiply this equation by 7, and it becomes $\frac{7}{29} = 0.24137931034482758620 \frac{10}{29}$, and then by substitution $\frac{1}{29} = 0.0344827586206896551724137931034482758620 \frac{10}{29}$, where every operation will at least double the number of figures found by the preceding operation. And this will be an easy expedient for converting division into multiplication in all cases. For this reciprocal of the divisor being thus found, it may be multiplied into the dividend to produce the quotient.”

17. Reduce $\frac{18}{99}$ to an equivalent decimal.
18. Reduce $\frac{27}{99}$ to an equivalent decimal.
19. Reduce $\frac{54}{99}$ to an equivalent decimal.
20. Reduce $\frac{63}{99}$ to an equivalent decimal.
21. Reduce $\frac{72}{99}$ to an equivalent decimal.
22. Reduce $\frac{148}{999}$ to an equivalent decimal.
23. Reduce $\frac{185}{999}$ to an equivalent decimal.
24. Reduce $\frac{259}{999}$ to an equivalent decimal.
25. Reduce $\frac{296}{999}$ to an equivalent decimal.
26. Reduce $\frac{370}{999}$ to an equivalent decimal.
27. Reduce $\frac{407}{999}$ to an equivalent decimal.
28. Reduce $\frac{481}{999}$ to an equivalent decimal.
29. Reduce $\frac{518}{999}$ to an equivalent decimal.
30. Reduce $\frac{592}{999}$ to an equivalent decimal.
31. Reduce $\frac{153846}{999999}$ to an equivalent decimal.
32. Reduce $\frac{230769}{999999}$ to an equivalent decimal.

PROBLEM II.

Any decimal of a superior integer being given, to determine its value in the known parts of that integer; or, which is the same thing, to express its value by inferior integers, their decimals, or both.

RULE.

RULE*.

1. Multiply the given decimal by the value of its integer in the next inferior order; and point off, from right to left, as many figures of the product as there were places in the given decimal.

2. Multiply the decimal last pointed off by the value of its integer, in the next inferior order, pointing off the same number of decimals as before; and thus continue the process to the lowest integer, or until the decimals cut off become all cyphers; then will the several numbers on the left of the separating points, together with the remaining decimal if any, express the required value of the given decimal.

EXAMPLES.

1. What is the value of .825l.? *Ans.* 16s. 6d.
2. What is the value of .528125l.? *Ans.* 10s. 6d. 3q.
3. What is the value of .964583l.? *Ans.* 19s. 3d. 2q.
4. What is the value of .8125s.? *Ans.* 9d. 3q.
5. What is the value of .8655l. *Ans.* 17s. 3d. 2.88q.
6. What is the value of .525 crown? *Ans.* 2s. 7d. 2q.
7. What is the value of .788889lb. troy? *Ans.* 9oz. 9dwts. 8gr.
8. What is the value of .46945lb. troy? *Ans.* 5oz. 12dwts. 16.032gr.
9. What is the value of .9375 cwt. *Ans.* 3q. 21lb.
10. What is the value of .79375 ton? *Ans.* 15cwt. 3qrs. 14lb.
11. What is the value of .68825 yard? *Ans.* 2q. 3'012n.
12. What is the value of .67625 mile? *Ans.* 5f. 16p. 2yd. 0ft. 7.2in.
13. What is the value of .6685 acre? *Ans.* 2r. 26p. 29yd. 0.36ft.

* Equivalent fractions of different integers are reciprocally as these integers, per Prob. 8th of Vulgar Fractions; hence the truth of the rule is manifest; for if 825 be the fraction of a pound sterling to be expressed in shillings, &c. the two integers are as 20 : 1; therefore, as 1 : 20 :: .825 : 16.5; the .5 is the fraction of a shilling to be expressed in pence; the integers are as 12 : 1; therefore as 1 : 12 :: .5 : 6. from whence it appears that .825l. = 16s. 6d.

14. What

14. What is the value of .787125 *tun*?

Ans. 3 *bbl.* 9 *gal.* 2.844 *pt.*

15. What is the value of .6171875 *quarter of corn*?

Ans. 4 *b.* 3 *p.* 1 *gal.* 4 *pints.*

16. What is the value of .8875 *month*?

Ans. 3 *w.* 3 *d.* 20 *h.* 24 *min.*

PROBLEM III.

Any number of inferior integers, or their decimals, being given, to express their value by the decimal of a superior integer.

RULE*.

1. Reduce the lowest number to a decimal of the next superior integer, per problem first, to which add the integers or decimals in that order, if any.

2. Reduce the resulting number to a decimal of the next superior integer, and so on to that of the integer mentioned; the decimal thus found will be equivalent to the inferior numbers given.

EXAMPLES.

1. Reduce 16s. 6d. to the decimal of a pound sterling.

Ans. .825 *l.*

2. Reduce 10s. 6d. 3q. to the decimal of a pound sterling.

Ans. .528125 *l.*

3. Reduce 19s. 3d. 2q. to the decimal of a pound sterling.

Ans. .964583 *l.*

4. Reduce 9d. 3q. to the decimal of a shilling.

Ans. .8125 *s.*

5. Reduce 17s. 3d. 2.88q. to the decimal of a pound sterling.

Ans. .8655 *l.*

6. Reduce 2s. 7d. 2q. to the decimal of a crown.

Ans. .525 *c.*

7. Reduce 9oz. 9dwt. 8gr. to the decimal of a pound troy.

Ans. .788889 *lb.*

8. Reduce 5oz. 12dwt. 16.032gr. to the decimal of a pound troy.

Ans. .46945 *lb.*

9. Reduce 3qrs. 21lb. to the decimal of a hundred weight.

Ans. .9375 *cwt.*

* This is only the reverse of Problem 2d, and needs no farther illustration.

10. Reduce 15cwt. 3qrs. 14lb. to the decimal of a ton.
Ans. .79375 ton.
11. Reduce 29. 3'012n. to the decimal of a yard.
Ans. .68825 yard.
12. Reduce 5f. 16p. 2yd. 0ft. 7.2in. to the decimal of a mile.
Ans. .67625 mile.
13. Reduce 2r. 26p. 29yd. 0.36ft. to the decimal of an acre.
Ans. .6685 acre.
14. Reduce 3bhd. 9gal. 2.844pts. to the decimal of a tun.
Ans. .787125 tun.
15. Reduce 4b. 3p. 1gal. 4pts. to the decimal of a quarter of corn.
Ans. .6171875 qr.
16. Reduce 3w. 3d. 20h. 24min. to the decimal of a month.
Ans. .8875 mon.

THE DOCTRINE OF CIRCULATING DECIMALS.

1. CIRCULATING DECIMALS are infinite converging series arising from the evolution of those ratios to which, no equivalents can be found, having their antecedents finite whole numbers, and their consequents pure powers of ten: they are divided into simple and multiple, and these again into pure and mixed.

2. Pure single circulates are those which only repeat single digits; as .666, &c. or .444, &c. and may be marked thus: .6, .4.

3. Pure multiple circulates are those which repeat several figures; as .684684, &c. or .586586, &c. and may be marked thus: .684, .586.

4. Mixed single circulates are those which consist of terminate or finite parts, and single repeating figures; as .58444, &c. or .642333, &c. and may be marked thus: .584, 6423.

5. Mixed multiple circulates are those which consist of terminate or finite parts, and several repeating figures; as .74586586, &c. or .48372372, &c. and may be marked thus: .74586, 48372.

6. Those parts which repeat are the proper circulates, and may be called repetends or circuli; as in .586, and .784325, the parts 6, and 325 being the proper circulates, may therefore be called repetends or circuli, the other parts being finite may be detached from the expressions.

7. If circulates begin and end at the same place, they are said to be similar and conterminous; but when they begin or end at different places, they are said to be dissimilar.

8. The equivalent vulgar fraction, or ratio, from which any circulate is extracted, may be called the radix of that circulate; by determining which the two problems immediately following are solved.

PROBLEM I.

Any pure circulate, whether simple or multiple, being given, to determine its radix.

RULE *.

Multiply the given circulate by a vulgar fraction, having for its numerator that power of ten, the index of which expresses the

* DEM. Put r = the required radix, c = the circulating part, and n = the number of places in c ; then $r - \frac{r}{10^n} = r \times \frac{10^n - 1}{10}$
 $= c, \therefore r = c \times \frac{10^n}{10^n - 1}$. Q. E. D.

Let (e. g.) $r = .142857, 142857, +, \&c.$ fine fine; then $n = 6$,
 and $r - \frac{r}{10^n} =$

$$\begin{aligned} &.142857, 142857 + \&c. \\ &- .000000, 142857 + \&c. = .142857 = c, \text{ and } r = c \times \frac{10^n}{10^n - 1}, = \\ &\frac{142857}{999999} = \frac{1}{7} \end{aligned}$$

COROLLARIES.

1. In every pure circulate, the whole repeating part, being continued ad infinitum, is equal to a vulgar fraction, of which the numerator is the repeating number, having the decimal point removed as many places to the right hand as there are places in the repetend, and the denominator an equal number of nines.

2. If any number N , consisting of n places, be multiplied by $\frac{10^n}{10^n - 1}$, and expanded into a series, it will become $N + \frac{N}{10^n} + \frac{N}{10^{2n}} + \frac{N}{10^{3n}} + \frac{N}{10^{4n}} +, \text{ fine fine, the total value of which will be to } N \text{ as } 10^n : 10^n - 1.$

3. If

the number of places in the circulus, and denominator the same number of nines; the product will be the radix, or equivalent vulgar fraction required.

EXAMPLES.

Required the radices, or least vulgar fractions equivalent to the following circulates ?

1.	_____	Ans. $\frac{1}{9}$.	18	_____	Ans. $\frac{2}{11}$.
2.	_____	Ans. $\frac{2}{9}$.	27.	_____	Ans. $\frac{3}{11}$.
3.	_____	Ans. $\frac{1}{3}$.	36.	_____	Ans. $\frac{4}{11}$.
4.	_____	Ans. $\frac{4}{9}$.	45.	_____	Ans. $\frac{5}{11}$.
5.	_____	Ans. $\frac{5}{9}$.	54.	_____	Ans. $\frac{6}{11}$.
6.	_____	Ans. $\frac{2}{3}$.	63.	_____	Ans. $\frac{7}{11}$.
7.	_____	Ans. $\frac{7}{9}$.	72.	_____	Ans. $\frac{8}{11}$.
8.	_____	Ans. $\frac{8}{9}$.	81.	_____	Ans. $\frac{9}{11}$.
9.	_____	Ans. $\frac{1}{1} = 1$.	90.	_____	Ans. $\frac{10}{11}$.
09.	_____	Ans. $\frac{1}{11}$.	037.	_____	Ans. $\frac{1}{11}$.

3. If any pure circulate be multiplied by as many nines (considered as decimals) as it contains places, the result will be the same number, complete and terminate; because $r \times \frac{10^n - 1}{10^n} = c$.

4. Any pure multiple circulate is equal to the sum of as many pure circulates of the same number of places, as there are significant figures in the circulus: for the circulate $\frac{142857}{999999} = \frac{100000}{999999} + \frac{400000}{999999} + \frac{2000}{999999} + \frac{800}{999999} + \frac{50}{999999} + \frac{7}{999999} = .100000 + .400000 + .00200000 + .000800000 + .0000500000 + .00000700000 = .142857$.

N. B. Some may perhaps consider the cyphers preceding the significant figures in the last five of the above circulates, as finite parts, because they do not enter the several circuli: but they are not properly finite parts; for there exist no real quantities to which they are equivalent: they are only indices pointing out the local values of the significant figures in the several series.

CIRCULATING DECIMALS.

97

•074.	—	<i>Ans.</i> $\frac{2}{27}$	•307692.	—	<i>Ans.</i> $\frac{4}{13}$
•148.	—	<i>Ans.</i> $\frac{4}{27}$	•384615.	—	<i>Ans.</i> $\frac{5}{13}$
•185.	—	<i>Ans.</i> $\frac{5}{27}$	•461538.	—	<i>Ans.</i> $\frac{6}{13}$
•259.	—	<i>Ans.</i> $\frac{7}{27}$	•538461.	—	<i>Ans.</i> $\frac{7}{13}$
•296.	—	<i>Ans.</i> $\frac{8}{27}$	•615384.	—	<i>Ans.</i> $\frac{8}{13}$
•370.	—	<i>Ans.</i> $\frac{10}{27}$	•692307.	—	<i>Ans.</i> $\frac{9}{13}$
•407.	—	<i>Ans.</i> $\frac{11}{27}$	•769230.	—	<i>Ans.</i> $\frac{10}{13}$
•481.	—	<i>Ans.</i> $\frac{12}{27}$	•846153.	—	<i>Ans.</i> $\frac{11}{13}$
•518.	—	<i>Ans.</i> $\frac{14}{27}$	•923076.	—	<i>Ans.</i> $\frac{12}{13}$
•592.	—	<i>Ans.</i> $\frac{15}{27}$	•0588235294117647.	<i>Ans.</i> $\frac{1}{17}$	
•629.	—	<i>Ans.</i> $\frac{17}{27}$	•1176470588235294.	<i>Ans.</i> $\frac{2}{17}$	
•703.	—	<i>Ans.</i> $\frac{10}{27}$	•1764705882352941.	<i>Ans.</i> $\frac{3}{17}$	
•740.	—	<i>Ans.</i> $\frac{20}{27}$	•2352941176470588.	<i>Ans.</i> $\frac{4}{17}$	
•814.	—	<i>Ans.</i> $\frac{22}{27}$	•2941176470588235.	<i>Ans.</i> $\frac{5}{17}$	
•851.	—	<i>Ans.</i> $\frac{23}{27}$	•3529411764705882.	<i>Ans.</i> $\frac{6}{17}$	
•925.	—	<i>Ans.</i> $\frac{25}{27}$	•4117647058823529.	<i>Ans.</i> $\frac{7}{17}$	
•962.	—	<i>Ans.</i> $\frac{26}{27}$	•4705882352941176.	<i>Ans.</i> $\frac{8}{17}$	
•076923.	—	<i>Ans.</i> $\frac{1}{13}$	•5294117647058823.	<i>Ans.</i> $\frac{9}{17}$	
•153846.	—	<i>Ans.</i> $\frac{2}{13}$	•5882352941176470.	<i>Ans.</i> $\frac{10}{17}$	
•230769.	—	<i>Ans.</i> $\frac{3}{13}$	•6470588235294117.	<i>Ans.</i> $\frac{11}{17}$	

PROBLEM II.

Any mixed circulate, whether simple or multiple, being given, to determine its radix.

RULE *.

Reduce the finite or terminate part to its least equivalent, per problems 1st and 2d of Vulgar Fractions, and the repetend, or

* That the value of the finite part added to that of the circulating part, should be equal to the value of both, is founded upon an axiom

K

too

or circulating part, per problem 1st of this; their sum, in lowest terms, will be the radix, or least equivalent Vulgar Fraction required.

EXAM-

too well known to need repetition: for if we put $\frac{a}{b}$ = the value of the finite part, r , c , and n as before; it is evident that $\frac{a}{b} + c \times \frac{10^n}{10^n - 1} = \frac{a \times 10^n - 1 + b c \times 10^n}{b 10^n - 1} = r$.

Let (e. g.) $r = 2\dot{1}4285\dot{7}$ +, &c. fine fine; then the terminate part = $\frac{2}{10} + \frac{1}{5}$, and the repetend or circulating part = $\frac{14285.7}{99999} = \frac{1}{70}$; therefore $\frac{1}{5} + \frac{1}{70} = \frac{3}{14} = r$.

COROLLARIES.

1. If the vulgar fraction $\frac{10^n}{10^n - 1}$ be multiplied by any mixed circulate, simple or multiple, and the numerator of the product made less by the finite part retaining its primitive value, the remaining expression will be equivalent to the mixed circulate: for suppose the value of a as well as c to be expressed in the decimal scale of notation, then b will vanish, and the above expression for the value of r , will become $= \frac{a + c \times 10^n - a}{10^n - 1}$.

Let (e. g.) $r = 2\dot{1}4285\dot{7}$, &c. as above, then, $2\dot{1}4285\dot{7} \times \frac{10}{99999} = \frac{214285.7}{99999}$, and $\frac{214285.7 - 2}{99999} = \frac{214285.5}{99999} = \frac{3}{14} = r$ as before.

If a vulgar fraction have any number of nines for its denominator, less than the number of significant figures in its numerator; that fraction is equal to a mixed circulate, of which the repetend has the same number of places as there are nines in the denominator; and its circulate value may be found from the converse of the above

process; (i. e.) multiply the given fraction by $\frac{10^n - 1}{10^n}$ = the reciprocal

of $\frac{10^n}{10^n - 1}$ and add the finite part to the product; the result will be

the

CIRCULATING DECIMALS.

99

EXAMPLES.

Required the radices, or least vulgar fractions equivalent to the following circulates?

·12̄.	—	<i>Ans.</i> $\frac{1}{90}$.	·763̄.	—	<i>Ans.</i> $\frac{42}{53}$.
·23̄.	—	<i>Ans.</i> $\frac{7}{30}$.	·872̄.	—	<i>Ans.</i> $\frac{48}{53}$.
·31̄.	—	<i>Ans.</i> $\frac{31}{90}$.	·981̄.	—	<i>Ans.</i> $\frac{54}{53}$.
·45̄.	—	<i>Ans.</i> $\frac{41}{90}$.	·0714285̄.	—	<i>Ans.</i> $\frac{1}{14}$.
·56̄.	—	<i>Ans.</i> $\frac{51}{90}$.	·2142857̄.	—	<i>Ans.</i> $\frac{3}{14}$.
·67̄.	—	<i>Ans.</i> $\frac{61}{90}$.	·3571428̄.	—	<i>Ans.</i> $\frac{5}{14}$.
·78̄.	—	<i>Ans.</i> $\frac{71}{90}$.	·6428571̄.	—	<i>Ans.</i> $\frac{9}{14}$.
·89̄.	—	<i>Ans.</i> $\frac{8}{10}$.	·7857142̄.	—	<i>Ans.</i> $\frac{11}{14}$.
·109̄.	—	<i>Ans.</i> $\frac{6}{53}$.	·9285714̄.	—	<i>Ans.</i> $\frac{13}{14}$.
·218̄.	—	<i>Ans.</i> $\frac{12}{53}$.	·03571428̄.	—	<i>Ans.</i> $\frac{1}{28}$.
·327̄.	—	<i>Ans.</i> $\frac{13}{53}$.	·10714285̄.	—	<i>Ans.</i> $\frac{3}{28}$.
·436̄.	—	<i>Ans.</i> $\frac{24}{53}$.	·17857142̄.	—	<i>Ans.</i> $\frac{5}{28}$.
·654̄.	—	<i>Ans.</i> $\frac{36}{53}$.	·32142857̄.	—	<i>Ans.</i> $\frac{9}{28}$.

the mixed circulate; (e. g.) let $\frac{2142855}{999999}$ be the given fraction to

which an equivalent circulate is required; then $\frac{2142855}{999999} \times \frac{9...6}{10^6} =$

·2142855, to which if 2, the finite part, be added, it will become ·2142857, the equivalent circulate required.

3. The circulating figures may be supposed to begin at any place in the repetend; because ·23452345, &c. fine fine is evidently

$$= \frac{2345}{9999} = \frac{2}{9999} = \frac{3452}{9999} = \frac{4523}{9999} = \frac{5234}{9999}.$$

4. The repeating figures of any circulating number may be considered as consisting of twice or thrice that number of figures, or any multiple thereof: for 6·84545, + &c. fine fine = 6·845 = 6·84545 = 6·8454545, &c.

5. From the third and fourth Corollaries it is evident that any number of dissimilar repetends may be made similar and conterminous; by performing which the next Problem is solved.

K 2

·39285714.

$\cdot 39285714$ —	<i>Ans.</i> $\frac{11}{8}$	$5\cdot 6076923$ —	<i>Ans.</i> $\frac{729}{148}$
$\cdot 4642871$ —	<i>Ans.</i> $\frac{13}{8}$	$11\cdot 2153846$ —	<i>Ans.</i> $\frac{729}{83}$
$\cdot 53571428$ —	<i>Ans.</i> $\frac{15}{8}$	$16\cdot 8230769$ —	<i>Ans.</i> $\frac{217}{138}$
$\cdot 60714285$ —	<i>Ans.</i> $\frac{17}{8}$	$22\cdot 4307692$ —	<i>Ans.</i> $\frac{145}{85}$
$\cdot 67857142$ —	<i>Ans.</i> $\frac{19}{8}$	$28\cdot 0384615$ —	<i>Ans.</i> $\frac{729}{28}$
$\cdot 82142857$ —	<i>Ans.</i> $\frac{23}{8}$	$56\cdot 076923$ —	<i>Ans.</i> $\frac{729}{13}$
$\cdot 89285714$ —	<i>Ans.</i> $\frac{25}{8}$	$112\cdot 153846$ —	<i>Ans.</i> $\frac{145}{13}$
$\cdot 97142857$ —	<i>Ans.</i> $\frac{27}{8}$	$168\cdot 230769$ —	<i>Ans.</i> $\frac{217}{13}$

PROBLEM III.

Any number of dissimilar repetends being given, to make them similar and conterminous.

RULE *.

Make them all begin at the same place, and consist of as many places as the least common multiple of all their several number of places contains units.

EXAMPLES.

Make the following circulates similar and conterminous.

Dissimilar.

$22\cdot 6\dot{5}4$
 $28\cdot 471428\dot{5}$
 $74\cdot 08\dot{3}$
 $42\cdot 5\dot{6}$
 $27\cdot 58\dot{3}$
 $68\cdot 345\dot{8}$

Similar and Conterminous.

$22\cdot 654545454545\dot{4}5$
 $28\cdot 471428571428\dot{5}7$
 $74\cdot 083333333333\dot{3}3$
 $42\cdot 566666666666\dot{6}6$
 $27\cdot 58358358358\dot{3}58$
 $68\cdot 345834583458\dot{3}4$

* The circulating figures may begin any where within the repetend, per Coroll. 3d, Prop. 11.—Hence they may all be made to commence at the same place; and the making them consist of as many places as the least common multiple of all their several number of places contains units, is nothing more than reducing their equivalent vulgar fractions to the least common denominator of the same form; or which shall all be expressed by the same figures; by which they are prepared for addition and subtraction.

PRO.

PROBLEM IV.

Any vulgar fraction, or ratio, being given, to determine the number of places in which its equivalent decimal will either terminate or circulate.

RULE*.

1. Divide the terms of the given fraction by their greatest com-

* 1. If any two numbers a and b be prime to each other, any power of the one will be prime to any power of the other, (i. e.) a^m prime to b^n . From whence it follows that any number prime to 10 will also be prime to all its powers; and therefore if 10, and its component factors 2 and 5, be expunged from the composition of any number, the remainder, if any, will measure no power of 10 by any terminate whole number; but if any number a be divided by any other number b which permits the division to be continued beyond $b-1$ places, the same remainder must return either then or before: therefore if any number n , having neither 2, 5, nor 10, in its composition, be made a divisor to 10^n , the quotient will either circulate in $n-1$ places or fewer, (i. e.) the remainder will either then or before become 1; but whenever the remainder becomes one, it is plain that if the dividend had been made less by that one, the division would then have terminated, and the dividend consisted of as many nines as there were units in the index of that power employed.

2. Again, if any two numbers a and b be prime to each other, a shall measure no multiple of b , and b no multiple of a less than $a \times b = p$; and since all their powers are prime to each other, a^m , and b^n , shall measure no multiples of each other less than $a^m \times b^n$; but $a^m \times b^n$, can measure no power of p less than that expressed by the higher index, whether m or n , suppose m ; then $a^m \times b^n$, measures pm by b^{m-n} ; but $a^m \times b^n$ is composed of as many factors a b and a as there are units in m : therefore if $am \times bn$ be divided as often as possible by a b , and then by a , the whole will vanish in m operations.

If now 2, 5, and 10 be substituted for a , b , and p ; and any whole numbers for m and n ; then $2^m \times 5^n$ will measure no power of 10 less than 10^m , which it measures by 5^{m-n} ; but if 5^{m-n} be a decimal, it must express the value of the ratio $\frac{10^0}{2^m \times 5^n} = 5^{\frac{m-n}{10^m}}$; therefore the quotient will contain as many places as there are units

common measure, and the new denominator by 2, 5, or 10, until the result be prime to these divisors.

units in m ; (e. g.) let $m = 4$, and $n = 2$, then $2^m \times b^n = 400$, and $10^m = 10000$, the quotient in whole numbers being 25; but if the

quotient must express the ratio of $1:400$, or $\frac{1}{400}$, it will then become

$\cdot 0025$, consisting of as many places as 400 has factors 10, and 2: for $2 \times 2 \times 10 \times 10 = 400$.

3. Also if there be any multitude of numbers a, b, c, d , &c. prime to each other, which circulate in m, n, u, v , &c. places respectively; and N be the least common dividend of m, n, u, v .—I say, that if the product $a \times b \times c \times d$, &c. be made a divisor, the quotient will circulate in N places.

For let $r \times m = x \times n = y \times u = z \times v = N$. It is evident that r periods of m , x periods of n , y periods of u , and z periods of v , will all terminate together in N places, if they begin together, which they be supposed to do, per Coroll. 3d, prob 2d; but they do not terminate sooner, since N is their least common dividend: therefore $a \times b \times c \times d$, &c. being made a divisor, the quotient will circulate in N places.

N. B. I hope the Author of a late publication will, in the next edition, correct an error which has either dropt from his pen, or the printer's fingers, in the solution of Ex. 1st to Case 4th in Reduction of Circulating Decimals: the Author's words are:

“Required to find whether the decimal equal to $\frac{210}{1120}$ be finite or infinite; and, if infinite, how many places that repetend will consist of?

First 10) $\frac{210}{1120} = \frac{21}{112}$, 2) $112 = 56 = 28 = 14 = 7$. Then 7) $\frac{99999}{142857}$;

and therefore the decimal is infinite, and the circulate consists of 6 places, beginning at the decimal point.”

The fraction $\frac{210}{1120} = \frac{3}{16} = \cdot 1875$, terminates in 4 places.

The denominator 112 will indeed, with all numerators prime to itself, give a circulate of 6 places; none of which however can commence before the 5th place from the decimal point, (i. e.) they will all consist of 6 places circulus, and 4 places finite part: for while the denominator continues the same, and the terms of the ratio coprimes, the number of places in both circulus and finite part will remain unvariable.

2. Divide a series of nines by the former result, until nothing remains, and the number of nines employed will be equal to the number of places in the circulus; which will commence after as many places as there were divisions by 2, 5, or 10.

3. If the whole denominator vanishes in dividing by 2, 5, or 10, the decimal will be finite, and terminate in as many places as there were divisions.

EXAMPLES.

Required the number of places in which the decimals equivalent to the following ratios will either circulate or terminate?

$\frac{1}{11}$ &c. $\frac{10}{11}$. All circulate in 2 places, without finite part.
 $\frac{1}{7}$ &c. $\frac{6}{7}$. All circulate in 6 places, without finite part.
 $\frac{1}{13}$ &c. $\frac{12}{13}$. All circulate in 6 places, without finite part.
 $\frac{1}{17}$ &c. $\frac{16}{17}$. All circulate in 16 places, without finite part.
 $\frac{1}{19}$ &c. $\frac{18}{19}$. All circulate in 18 places, without finite part.
 $\frac{1}{23}$ &c. $\frac{22}{23}$. All circulate in 22 places, without finite part.
 $\frac{1}{14}$, $\frac{3}{14}$, $\frac{5}{14}$, $\frac{9}{14}$, $\frac{11}{14}$, $\frac{13}{14}$. All circulate in 6 places, with 1 place finite part.

$\frac{1}{28}$, $\frac{3}{28}$, $\frac{5}{28}$, $\frac{9}{28}$, $\frac{11}{28}$, $\frac{13}{28}$, $\frac{15}{28}$, $\frac{17}{28}$, $\frac{19}{28}$, $\frac{23}{28}$, $\frac{25}{28}$, $\frac{27}{28}$. All circulate in 6 places, with 2 places finite part.

$\frac{2}{11}$, $\frac{25}{11}$, and all prime terms to $\frac{111}{11}$ inclusive, circulate in 6 places, with 4 places finite part.

$\frac{110}{120} = \frac{11}{12} = \frac{3}{4}$, will terminate in 4 places, and likewise $\frac{1}{12}$

$\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$, $\frac{11}{16}$, $\frac{13}{16}$, $\frac{15}{16}$ will terminate in 4 places; $\frac{1}{84}$, $\frac{3}{84}$, $\frac{5}{84}$, &c.,

$= \frac{1}{28}$, $\frac{3}{28}$, $\frac{5}{28}$, &c. will, while the numerators are primes to 64,

and the fractions proper, all terminate in 6 places from the decimal point.

ADDITION OF CIRCULATING DECIMALS.

PROBLEM V.

Any multitude of circulating numbers being given, to determine their sum.

RULE.

Make them all similar and conterminous, per Prob. 3d; increase the right-hand column by as many units as would be carried thereto if the several circuli were continued forward; then

104 ADDITION OF CIRCULATING DECIMALS.

then add as in common numbers, and the sum total will be that required.

EXAMPLES.

Required the sum of the following circulates ?

24.875	24.87587587587	58
48.32	48.323232323232	32
428.76	428.766666666666	66
36.216	36.216216216216	62
26.6184	26.618461846184	84
47.083	47.083333333333	33
84.41	84.414141414141	41
98.4142	98.414241424142	42

794.71216909979286

398.76432
973.25864
29.5863
786.394378
980.43758
246.5832
78.89
586.32456
371.225
384.6

735.43257
849.64523
786.7854
874.3246
798.586

3578.69325
6789.23768
4586.35872
2984.278694
5837.1073589
8537.867
9873.87586
3942.374
6732.4325
7867.083

3258.6735849
2374.587634
6758.32567
7864.8732
8759.467

99.986

99·986̄
228·083̄
678·7345678̄
957·326̄
248·98̄

673·4789657̄
73·5684321̄
842·32586̄
374·2345̄
739·624̄
851·08̄
578·8̄
428·78325̄
710·246837̄
897·3456789̄

3842·58̄
7649·4̄
9867·45678̄
4237·5863̄
8745·378̄

2587·42653̄
8674·3258̄
7867·245̄
8496·75̄
9246·8̄
2924·8735̄
3876·437̄
3248·37̄
5864·8̄
6743̄

SUBTRACTION OF CIRCULATING DECIMALS.

PROBLEM VI.

Any two circulating numbers being given, to determine their difference.

RULE.

Make them fimilar and conterminous, as in addition; and if the circulus of the subducend be greater than that of the minuend, increafe its right-hand figure by one; subtract as in common numbers, and the difference will be that required.

EXAMPLES.

Required the difference of the following circulates ?

From 785·34587̄
Take 329·582645̄

785·34587587587587587̄ 58
329·58264582645826458̄ 26
455·76323004941761129̄

From

106 MULTIPLICATION OF CIRCULATING DECIMALS.

From $809\cdot50724$

Take $457\cdot4682$

From $549\cdot35746$

Take $37\cdot08542$

From $7539\cdot0327$

Take $422\cdot79467$

From $385\cdot2375$

Take $42\cdot586$

From $783\cdot27$

Take $123\cdot95672$

MULTIPLICATION OF CIRCULATING DECIMALS.

P R O B L E M VII.

Any two circulating numbers being given, to determine their product.

R U L E.

1. Convert both factors into their equivalent vulgar fractions, and find their product.

2. Reduce the vulgar fraction expressing their product to an equivalent decimal, continuing the process until the law of continuation is discovered; and the result will be that required.

E X A M P L E S.

Required the products of the following circulates ?

$$\cdot12 \times \cdot23 \quad \text{Ans. } \cdot02851$$

$$\cdot34 \times \cdot45$$

$$\cdot89 \times \cdot109$$

$$\cdot0714285 \times \cdot1428571$$

$$\cdot2142857 \times \cdot3571428$$

$$\cdot7857142 \times \cdot9285714$$

$$\cdot03571428 \times \cdot17857142$$

$$22\cdot4307692 \times 16\cdot8230769$$

$$11\cdot2153846 \times 28\cdot0384615$$

$$56\cdot076923 \times 33\cdot7258736$$

$$112\cdot3986457 \times 46\cdot28643967$$

$$442\cdot7983567 \times 22\cdot35873425$$

$$837\cdot256789 \times 428\cdot584937$$

$$999\cdot5732456 \times 866\cdot789456$$

DIVISION OF CIRCULATING DECIMALS.

P R O B L E M VIII.

Any two circulating numbers being given, to determine their quotient.

R U L E.

1. Convert both divisor and dividend into their equivalent vulgar fractions, and find their quotient.

2. Reduce the vulgar fraction, expressing their quotient, to an equivalent decimal, continuing the process until the law of continuation is discovered, and the result will be that required.

E X A M P L E S.

$$\cdot23 \div \cdot12$$

$$\text{Ans. } 1\cdot90$$

$$\cdot45 \div \cdot34$$

$$\cdot109 \div \cdot89$$

$$\cdot1428571 \div \cdot0714285$$

$$\cdot3571428 \div \cdot2142857$$

$$\cdot9285714 \div \cdot7857142$$

$$\cdot17857142 \div \cdot03571428$$

$$16\cdot8230769 \div 22\cdot4307692$$

$$28\cdot0384615 \div 11\cdot2153846$$

$$56\cdot076923 \div 33\cdot7258736$$

$$112\cdot3986457 \div 46\cdot28643967$$

$$442\cdot7983567 \div 22\cdot35873425$$

$$837\cdot256789 \div 428\cdot584937$$

$$999\cdot5732456 \div 866\cdot789456$$

ALL-

ALLIGATION.

ALLIGATION is the mode of mixing several simples of different qualities, so that the composition may be of a middle quality, or value; and is commonly distinguished into two principal parts, denominated Alligation Medial, and Alligation Alternate.

ALLIGATION MEDIAL.

ALLIGATION MEDIAL is the mode of determining the rate of the compound, from having the rates and quantities of the several simples given: which determination may be easily obtained by the following

RULE.

Multiply each quantity by its rate; then divide the sum of the products by the sum of the quantities; and the quotient will be the required rate of the compound.

This rule may be expressed by the following analogy:

As the sum of all the quantities :
Is to the sum of all their rates, or values ::
So is any part of the mixture :
To the mean rate, or value of that part.

EXAMPLES.

1. A tea dealer made a composition consisting of 5lb. at 4s. 6lb. at 5s. 8lb. at 6s. 10lb. at 8s. and 11lb. at 12s. per lb. what is one pound of the composition worth? *Ans.* 7s. 9d. per lb.

2. A wine merchant mingles 14 gallons of Mountain wine at 8s. per gall. with 12 gallons at 6s. 2d. 10 gall. of Sherry at 7s. 20 gall. of white wine at 4s. and 8 gall. of Canary at 10s. per gallon: how may he sell the mixture per gallon?

Ans. 6s. 6d. per gall.

3. A composition being made of 12 bushels of wheat at 6s. 1d. per bushel, 20 bushels of oats at 5s. 16 bushels of barley at 4s. and 14 bushels of rye at 3s. per bushel: what is the composition worth per bushel? *Ans.* 4s. 6d. per bushel.

4. Having melted together 12 ounces of gold of 22 caracts fine, 14 oz. of 21, 10 oz. of 20, and 8 oz. of 18 caracts fine: I would know the fineness of the composition? *Ans.* 20½ caracts fine.

ALLIGATION ALTERNATE.

ALLIGATION ALTERNATE is the mode of determining what quantity of each of the simples, the rates of which are given, will compose a mixture of a given rate. This is the reverse of alligation medial: and therefore the two will mutually prove each other.

RULE *.

1. Write the rates of the simples in a column under each other.
2. Connect, or link with a continued line, the rate of each simple which is less than that of the compound, with one or any number of those which are greater than the compound; and each greater rate with one or any number of the less.
3. Write the difference between the mixture rate and that of each of the simples, opposite the rates with which these are linked.
4. Then if only one difference stand against any rate, it will be the quantity belonging to that rate; but if there be several, their sum will be the quantity.

EXAMPLES.

1. A merchant would mix wines at 16s. 18s. 22s. and 24s. per gallon, so that the whole may be worth 20s. per gallon: what quantity of each must be taken?—*Ans.* 1 of each, or equal quantities of each, let the number be what it will.

2. How much corn at 30s. 44s. 48s. and 56s. per quarter, must be mixed together, so that the compound may be worth 46s. per quarter?—*Ans.* 12 at 30s. 12 at 44s. 18 at 48s. and 18 at 56s.; or any quantity of each in the same proportion.

3. How much gold at 22, 21, 20, and 18 caracts fine, must be mixed together, so that the composition may be of 20½ caracts fine?—*Ans.* 3 at 22, 3 at 21, 2 at 20, and 2 at 18 caracts fine.

4. How much tea at 4s. 5s. 6s. 8s. and 12s. per lb. must be mixed together, so that the composition may be worth 7s. 9d. per lb.?—*Ans.* 4·25 lb. at 4s. 25 lb. at 5s. 4·25 lb. at 6s. 2·75 lb. at 8s. and 5·5 lb. at 12s. per lb.

When the price or rate of each simple is given, and also that of the composition, as before, but one of the simples limited to a given quantity, the solution may be obtained by the following

RULE.

1. Take the difference between the mean rate, and that of each simple, as before. Then,

* The reason of the rule given for alligation medial is plain: because, the value of the whole must be equal to the sum of the values of all its parts.

2. As the difference of that simple, the quantity of which is given
 Is to the assigned quantity of that simple : :
 So is any other of the differences found : :
 To the corresponding quantity required.

EXAMPLES.

1. How much wine at 6s. 7s. 8s. and 10s. per gallon must be mixed with 10 gallons at 5s. per gallon, so that the mixture may be worth 7s. 6d. per gallon?—*Ans.* 2 at 6s. 2 at 7s. 8 at 8s. and 10 at 10s. per gallon.

2. How much gold of 14, 16, 20, and 22 caracts fine, must be mixed with 6 ounces of 18 caracts fine, so that the composition may be of 19 caracts fine?—*Ans.* 18 oz. of 14, 6 oz. of 16, 24 oz. of 20, and 30 oz. of 22 caracts fine.

When the rates of the several simples are given, and also that of the compound, but the whole composition limited to a given quantity, the solution may be obtained by the following

RULE.

1. Take the difference between the mean rate, and that of each simple, as before. Then,
 2. As the sum of all these differences thus found : :
 Is to the whole quantity of the composition given : :
 So is each particular difference : :
 To its particular quantity required.

* It is easy to see that questions belonging to alligation alternate are unlimited, and therefore those that are brought to exercise rule 1st, where there is no limitation whatever, will admit of an infinite number of answers; and of all the above examples, the last only is truly limited. That rule 1st brings out one answer agreeable to the conditions of the proposition, is evident—thus:

Let r, n, u, v , be given rates, of which r and n are greater than m , the mean rate, but u and v less.

$$\text{Then } m \left[\begin{array}{c} r \\ n \\ u \\ v \end{array} \right] \begin{array}{c} m-v \\ m-u \\ n-m \\ r-m \end{array} \text{ the quantities required,}$$

which multiplied by their respective rates give $m-v \cdot r + m-u \cdot n + n-m \cdot u + r-m \cdot v = m \cdot r - v \cdot m + n \cdot m - u \cdot m$. The other two rules are evident from this, and the principles of proportion.

FALSE POSITION.

XII

EXAMPLES.

1. A vintner hath wines at 6s. at 8s. at 10s. at 12s. and at 14s. per gallon : what quantity of each must he take to make 140 gallons worth 10s. 6d. per gallon? — *Ans.* 35 at 6s. 15 at 8s. 15 at 10s. 30 at 12s. and 45 gallons at 14s. per gallon.

2. If 10.35 oz. 5.85 oz. 4.79 oz. 4.4 oz. and 3.86 oz. be the respective weights of a cubic inch of gold, silver, copper, brass, and tin : what quantity of each must I take to make a cubic foot, the weight of which shall be 720lb. ?

<i>Ans.</i>	Gold	218.88	<i>Cubic Inches.</i>
	Silver	155.52	
	Copper	163.20	
	Brass	163.20	
	Tin	1027.20	

3. Suppose the crown of Hiero, King of Syracuse, to have weighed 12lb. ; and let the specific gravities of gold, copper, and water, be as 18, 8, and 1 respectively. Then, the water expelled by the gold crown would only be 8 oz. but the water expelled by the copper crown would be 18 oz. ; and suppose the quantity of water expelled by the compounded crown to have been 12 oz. it is required from thence to determine the quantity of gold and of copper in the compounded crown ?

Ans. 7.2lb. of gold, and 4.8lb of copper.

FALSE POSITION.

FALSE POSITION is the mode of discovering numbers or quantities required, by substituting known numbers or quantities in their place ; and proceeding with these according to the conditions of the proposition.

POSITION is applied with success to those problems, the solutions of which cannot be obtained by a direct process ; and the solutions obtained by this method will be accurately true if the numbers or quantities required do not ascend above the first power ; but if otherwise they will only be approximations.

When some given function of the first power of the quantity required is equal to a quantity given, then it is evident that the results will be proportional to the suppositions : and therefore, one supposition only being necessary, the question is said to belong to single position. But if the conditions of the proposition require that the quantity sought shall either be increased or diminished by a quantity given, two suppositions become necessary, and the errors will then be proportional to the difference

ference between the true number and each supposition: hence the question is said to belong to double position; although for the most part they may be solved by one supposition only. Questions belonging to single position may be easily solved by the following

R U L E.

1. Assume any number at pleasure, with which proceed as if it were the number required.

2. Then, if the result be equal to that given, the number assumed is that required; but if not, it will be—As the result of the operation is to that given, so is the number assumed to that required.

E X A M P L E S.

1. What number is that which, multiplied by 7, and the product divided by 8, the quotient may be 512? *Ans.* 728.

2. A's age is triple of B's, and B's age is quadruple of C's, and the sum of all their ages is 136 years: what is the age of each? *Ans.* A's=96, B's=32, and C's=8.

3. A person being asked his age, said, if $\frac{2}{3}$ of the years I have lived be multiplied by 6, and $\frac{2}{3}$ of them be added to the product, the sum will be 249; what was his age? *Ans.* 60 years.

4. Four merchants, A. B. C. and D. bought a ship, which cost a sum unknown, of which sum A. paid one half, B. one fourth, C. one sixth, and D. one twelfth; moreover it appeared that $\frac{1}{2}$ the money paid by A. and D. was 50l. more than $\frac{2}{3}$ of that paid by B. and C.: required the cost of the ship, and the money paid by each.—*Ans.* The ship cost 3600l. of which A. paid 1800l. B. 900l. C. 600l. and D. 300l.

5. A shepherd being asked the number of sheep in his flock, replied, that the sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of his flock exceeded the whole by 60; required the number of sheep in the flock?

Ans. 720 sheep.

Questions belonging to double position may be solved by the following

R U L E *.

1. Assume any two numbers at pleasure, and proceed with each

* The reason of the first rule is evident: for let x = the number required, m, n, u, v , &c. any given coefficients, and a = any number whatever, assumed at pleasure.

$$\text{Then, } \frac{mx}{n} \pm \frac{ux}{v} \pm \&c. : x :: \frac{ma}{n} \pm \frac{ua}{v} \pm \&c. : a.$$

The

each according to the conditions of the proposition, as if it was the true number required ; and find how much the results are different from that in the proposition.

2. Multiply each of the errors by the contrary supposition, and find the sum and difference of these products.

3. If the errors are of the same affection, divide the difference of the products by the difference of the errors ; but if the errors are of different affections, divide the sum of the products by the sum of the errors ; and the quotient will be that required.

E X A M P L E S .

1. What number is that which, being multiplied by 12, the product increased by 24, and the sum divided by 8, the quotient will be 36 ?

Anf. 22.

2. A workman was hired for 40 days, at 2s. 6d. per day for every day he worked ; but with this condition, that for every day he played he should forfeit 1s. Now it so happened, that upon the whole he had 3l. 12s. to receive ; how many of the days did he work, and how many did he play ?

Anf. He worked 32, and played 8.

3. A son asking his father how old he was, receives the following answer : Your age is now $\frac{1}{4}$ of mine ; but five years

The second rule is founded upon the following hypothesis :— That the first error is to the second, as the difference between the true and first supposed number is to the difference between the true and second supposed number. That this is true, is thus manifest :

$$\text{Let } \frac{mx}{n} \pm a = b, \frac{mu}{n} \pm a = c, \text{ and } \frac{mv}{n} \pm a = d; \text{ then } \frac{mx}{n} \pm a \propto \frac{mu}{n} \pm a : \frac{mv}{n} \pm a :: x \propto u : x \propto v.$$

This being demonstrated, let A and B be any two results produced from the assumptions a and b by similar operations ; and let x be the required number from which N is produced by a similar operation ; also let $N \propto A = m$, and $N \propto B = n$.

Then, $m : n :: x - a : x - b \therefore x = \frac{mb - na}{m - n}$, when m and n are either both plus or both minus ; but when the one is plus and the other minus, the expression will become $x = \frac{mb + na}{m + n}$; which is agreeable to the rule.

ago your age was only $\frac{1}{3}$ of mine at that time. What are their ages?

Ans. The father 80, and the son 20.

4. Two persons A. and B. lay out equal sums of money in trade; A. gains 200l.; but B. loses 80l. and A.'s money is now double of B.'s; what did each lay out?

Ans. 360l.

5. A gentleman has two horses of considerable value, and a saddle worth 50l.; now, if the saddle be put on the back of the first horse, it will make his value double that of the second; but if it be put on the back of the second, it will make his value triple that of the first: what is the value of each horse?

Ans. The first 30l. and the second 40l.

6. There is a fish, the head of which is 9 inches long, and his tail is as long as his head and half his body, and his body is as long as his tail and his head: what is the whole length of the fish?

Ans. 72 in. = 6 feet.

N. B. It is easy to see how these questions may be solved by one supposition only. If we take (*e. g.*) the fifth, and inquire for the price of the first horse, it is plain, from the data, that we have only to find that number which, if multiplied by 3, and $\frac{1}{2}$ the number sought subtracted from the product, the remainder shall be equal to 75; and so on for any other.

I N V O L U T I O N *.

INVOLUTION is the mode of raising any given number to a certain power or dignity proposed. The mode of operation is the same as in Multiplication, save only—in that, the factors may be all different; in this, they are all the same: therefore a power or dignity is a number composed of equal factors, and the number of these factors is called the index of that power.

This being so perfectly easy, a rule is deemed unnecessary.

E X A M P L E S.

1. What is the second power of 8372.58?

2. What is the second power of 6842.543?

* The powers of numbers may be represented by a vinculum or bond drawn over them, with the index of the power at its extremity: thus $\overline{5864^2}$, $\overline{3258^3}$, $\overline{4586^4}$, represent the 2d, 3d, and 4th powers of these numbers respectively; and roots may be represented in like manner by reciprocal indices, thus $\overline{642^{\frac{1}{2}}}$, $\overline{586^{\frac{1}{3}}}$, $\overline{458^{\frac{1}{4}}}$, represent the square, cube, and biquadrate roots of these numbers respectively.

3. What

I N V O L U T I O N.

115

3. What is the third power of 586.25 ?
4. What is the third power of 4685.125 ?
5. What is the fourth power of 25.683 ?
6. What is the fourth power of 486.358 ?
7. What is the fifth power of 327.43 ?
8. What is the fifth power of 432.15 ?
9. What is the sixth power of 584.2 ?
10. What is the sixth power of 28.64 ?
11. What is the seventh power of 54.05 ?
12. What is the seventh power of 234.5 ?
13. What is the eighth power of 542.5 ?
14. What is the eighth power of 250.4 ?

TABLE

TABLE OF POWERS.

1st power	2	3	4	5	6	7	8	9
2d power	4	9	16	25	36	49	64	81
3d power	8	27	64	125	216	343	512	729
4th power	16	81	256	625	1296	2401	4096	6561
5th power	32	243	1024	3125	7776	16807	32768	59049
6th power	64	729	4096	15625	46656	117649	262144	531441
7th power	128	2187	16384	78125	279936	823543	2097152	4782969
8th power	256	6561	65536	390625	1679616	5764801	16777216	43046721
9th power	512	19683	262144	1953125	10077696	40353607	134217728	387420489
10th power	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
11th power	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
12th power	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481

EVO.

E V O L U T I O N.

EVOLUTION is the mode of extracting a certain proposed root from any given number.

A GENERAL PREPARATION*.

From the decimal point divide the number both ways into

* The general method of preparation is founded upon the following

THEOREM. If N be any composite number, n the number of component factors, and m the number of places in all the factors; I say the number of places in N is not greater than m , and not less than $m - n + 1$.

DEMON. In the first place let the several factors be the greatest possible, containing the same number of places; (i. e.) let all the factors consist of nines, and let r, s, u, v , &c. be their respective number of places; then $9^r \times 9^s \times 9^u \times 9^v$, &c. to n number of factors $= N =$ a number less than $9^r \times 10^s \times 10^u \times 10^v$, &c. to n number of factors; but the number of places in $9^r \times 10^s \times 10^u \times 10^v$, &c. to n number of factors is $= r + s + u + v + \&c.$ to n number $= m$; therefore the number of places in N is not greater than m .

Again, let the several factors be the least possible, containing the same number of places; (i. e.) let all the factors be pure powers of 10, the indices of which let be r, s, u, v , &c. respectively; then $10^r \times 10^s \times 10^u \times 10^v$, &c. to n number of factors $= N$; the number of places in which is $=$ the sum of the indices $+ 1$; but the number of places in all the factors $=$ the sum of the indices $+ n = S + n = m$; therefore $S + 1$, or the number of places in $N = m - n + 1$; and cannot be less; but it has also been proved that the number of places in N cannot be greater than m ; therefore if N be any composite number, &c. Q. E. D.

COROLLARIES.

1. A square number cannot have more places of figures than double the places of the root, and at least but one less.
2. A cube number cannot have more places of figures than triple the places of the root, and at least but two less.
3. A biquadrate number cannot have more places of figures than quadruple the places of the root, and at least but three less.
4. If any number N consist of m places, the number of places in N^n is not greater than $m \times n$, and not less than $m - 1 \times n + 1$.
From whence it is manifest,
5. That there will always be as many places in the root as there are periods in the power.

periods,

periods, containing as many places each as there are units in the index of that power the root of which is to be extracted; and if the right hand period is deficient in number, supply that deficiency by annexing cyphers.

TO EXTRACT THE SQUARE ROOT.

R U L E.

1. The number being prepared, find the greatest square in the left-hand period, and set its root on the right-hand of the given number, as a quotient in division.
2. Subtract the square thus found from the said period, and to the remainder annex the following period, for a dividend.
3. Double the above-mentioned root for a divisor; find how often it is contained in the dividend, exclusive of the units place; and set the result both in the quotient and divisor.
4. Subtract the product of this quotient figure, and the divisor thus augmented, from the dividend, and to the remainder annex the next period for a new dividend.
5. Find a new divisor as before, by doubling the figures already in the quotient; and from these find the next figure in the root, and so on until the whole is finished.

E X A M P L E S.

1. What is the square root of 695556? *Ans.* 834.
2. What is the square root of 575170.56? *Ans.* 758.4.
3. What is the square root of 788988.0625? *Ans.* 888.25.
4. What is the square root of 999500.0625? *Ans.* 999.75.
5. What is the square root of 719925.098256? *Ans.* 848.484.

TO EXTRACT THE CUBE ROOT.

R U L E I*.

1. The number being prepared, find the greatest cube in the

* The reason of this and the following Rule will appear from general or algebraic examples.

Suppose the number $N = a + b + c$; $N^2 = a^2 + 2ab + b^2 + 2ac + 2bc + c^2$. If now the root be required, the process will be as follows:

$$\begin{array}{r}
 a \overline{) a^2 + 2ab + b^2 + 2ac + 2bc + c^2} \\
 \underline{a^2} \\
 2a + b \overline{) 2ab + b^2} \\
 \underline{2ab + b^2} \\
 2a + 2b + c \overline{) 2ac + 2bc + c^2} \\
 \underline{2ac + 2bc + c^2} \\
 0
 \end{array}$$

Again,

the left hand period, and set the root on the right hand of the given number, as a quotient in division.

2. Subtract the cube thus found from the said period, and to the remainder annex the following period for a dividend.

Again, let $N = a + b + c$, as before; $N^3 = a^3 + 3a^2b + 3ab^2 + b^3 + 3a^2c + 6abc + 3b^2c + 3ac^2 + 3bc^2 + c^3$, and the root may be found by the following process:

$$\begin{array}{r}
 a^3 + 3a^2b + 3ab^2 + b^3 + 3a^2c + 6abc + 3b^2c + 3ac^2 + 3bc^2 + c^3(a+b+c) \\
 \hline
 3a^2 \cdot \quad 3a^2b + 3ab^2 + b^3 \\
 \hline
 3a^2b + 3ab^2 \\
 \hline
 + b^3 \\
 \hline
 3a^2b + 3ab^2 + b^3 \\
 \hline
 + 3a^2c + 6abc + 3b^2c + 3ac^2 + 3bc^2 + c^3 \\
 \hline
 3a^2c + 6abc + 3b^2c \\
 \hline
 + 3ac^2 + 3bc^2 \\
 \hline
 + c^3 \\
 \hline
 3a^2c + 6abc + 3b^2c + 3ac^2 + 3bc^2 + c^3
 \end{array}$$

N.B. The reason why the learner is directed to multiply the square of the quotient by 300, and that of the last found figure by 30, is only to keep the several products in their proper place; without which the tyro is apt to arrange them wrong.

3. Mul.

Again,

3. Multiply the square of the above-mentioned root by 300, and make the result a divisor ; by which divide the above dividend, and annex this quotient to the former.

4. Draw a line under the above-mentioned dividend, and write the following products below (i. e.) 1st, the product of the divisor and last quotient figure ; 2d, 30 times the square of the last quotient figure, multiplied by the former ; 3d, the cube of the last found quotient figure : take the sum of these three from the dividend, and to the remainder adjoin the next period for a new dividend.

5. Multiply the square of the whole quotient by 300, and make the result a new divisor, with which proceed as before, and so on until the whole is finished.

EXAMPLES.

	(1)	
	$\begin{array}{r} 654876557504 \\ 512 \\ \hline 142876 \\ 115200=6 \times 19200 \\ 8640=6^2 \times 8 \times 30 \\ 216=6^3 \\ \hline 124056 \\ 18820557 \\ 17750400=8 \times 2218800 \\ 165120=8^2 \times 86 \times 30 \\ 512=8^3 \\ \hline 17916032 \\ 904525504 \\ 904108800=4 \times 226027200 \\ 416640=4^2 \times 868 \times 30 \\ 64=4^3 \\ \hline 904525504 \\ \dots\dots\dots \end{array}$	$\begin{array}{l} 8684 \\ \\ \\ \\ \\ \\ \\ \\ \end{array}$
$8^2 \times 300 = 19200$		
$\overline{86}^2 \times 300 = 2218800$		
$\overline{868}^2 \times 300 = 226027200$		

- | | |
|--|-------------------|
| 2. What is the cube root of 48228544 ? | <i>Ans.</i> 364. |
| 3. What is the cube root of 146363183 ? | <i>Ans.</i> 527. |
| 4. What is the cube root of 1170905464 ? | <i>Ans.</i> 1054. |
| 5. What is the cube root of 216432288064 ? | <i>Ans.</i> 6004. |
| 6. What is the cube root of 3086626816 ? | <i>Ans.</i> 1456. |
| | 7. What |

7. What is the cube root of 7847380942848? *Ans.* 19872.

8. What is the cube root of 145220537353515625? *Ans.* 525625.

R U L E II*.

1. Find by trials the nearest rational cube to that given, and call it the assumed cube.

2. Then, twice the assumed cube added to that given, will be to twice the given cube added to that assumed, as the root of the assumed cube is to the root required, nearly. Or, as the first sum is to the difference of the given and assumed cube, so is the assumed root to the difference between the root assumed and that required, nearly.

3. Again, by taking the cube of the root found for a new assumed cube, and repeating the operation, a new root will be obtained to a still greater degree of exactness.

E X A M P L E S.

1. What is the cube root of 7854?

2. What is the cube root of 31416?

* This rule is only a particular case of an universal formula (discovered by one of the first mathematicians of the present age) for extracting all roots whatever, and includes all the rational formulæ of M. De Lagny and Dr. Halley. It is not only simple and easy to be remembered, but also very accurate: for it converges extremely fast, as will easily appear by expressing the rule in general terms.

Let N = the given number, a^3 = the assumed cube, and x = the correction. Then, $N + 2a^3 : 2N + a^3 :: a : \frac{2N + a^3}{N + 2a^3} a = a + x$;

which per reduction gives $N = a^3 + 3a^2x + 3ax^2 + x^3 + \frac{x^4}{2a}$,

and by neglecting the two last terms $+ x^3 + \frac{x^4}{2a}$, as being very small, there remains the complete cube $N = a^3 + 3a^2x + 3ax^2 +$

$x^3 \therefore N^{\frac{1}{3}} = a + x$ very nearly, if x is small; and by repeating the

operation with $a + x^3$ for the assumed cube, the root may be obtained to a still greater degree of accuracy.

M

3. What

3. What is the cube root of 12566 ?
4. What is the cube root of 20000 ?
5. What is the cube root of 75863 ?
6. What is the cube root of 52367 ?
7. What is the cube root of 888666 ?
8. What is the cube root of 787878 ?
9. What is the cube root of 10000000 ?
10. What is the cube root of 69432.58 ?
11. What is the cube root of 8686.4325 ?
12. What is the cube root of 953786.58132 ?
13. What is the cube root of 858678.32597632 ?
14. What is the cube root of 58679456789.5278978 ?
15. What is the cube root of 496786321058.2896783765 ?
16. What is the cube root of 678943256735586.3789458396 ?

TO EXTRACT ANY ROOT WHATEVER.

R U L E *.

Let N be the given power, or number, the root of which is to be extracted, n the index of that power, and a^n the assumed power.

Then,

* This rule (beyond which nothing can either be desired or hoped for, while our calculus stands on the present basis) is the invention of Dr. Charles Hutton ; it is published in his Arithmetic, p. 124, and demonstrated in a most general and satisfactory manner in his Tracts Mathematical and Philosophical. In pages 46 and 47 the Doctor investigates two separate theorems, differing from the truth by nearly equal small quantities, the one in excess and the other in defect, and by combining these two he obtains this general formula

formula $\frac{n+1 \cdot N + n-1 \cdot a^n}{n-1 \cdot N + n+1 \cdot a^n} a = N^{\frac{1}{n}} = a+x$ very nearly. But in page

48 the Doctor investigates the same theorem in a still more general manner ; and although I have not his leave, yet I hope I shall not offend by transcribing it : as it may perhaps be a stimulus to some to purchase these tracts, by the perusal of which they will find themselves amply rewarded both for time and money. The Author's words are—" But the third theorem may be investigated in a more general way, thus: Assume a quantity of this form $\frac{pN+qN}{qN+pN}$

a , with coefficients p and q to be determined from the pro-

cess ;

Then, as the sum of $n-1$ times N and $n+1$ times a^n , is to the sum of $n+1$ times N and $n-1$ times a^n , so is a , the assumed root, to the required root nearly.

Or, as half the first sum is to the difference between the given

cess; the other letters N, a, n , representing the same things as before; then divide the numerator by the denominator, and make the quotient equal to $a+x$, so shall the comparison of the coefficients determine the relation between p and q required. Thus,

$$pN+qa^n = \overline{p+q} \cdot a^n + pna^{n-1}x + pn \cdot \frac{n-1}{2} a^{n-2}x^2 + \&c.$$

$$qN+pa^n = \overline{p+q} \cdot a^n + qna^{n-1}x + qn \cdot \frac{n-1}{2} a^{n-2}x^2 + \&c.$$

then dividing the former of these by the latter, we have

$$\frac{pN+qa^n}{qN+pa^n} a \text{ or } a+x = a + \frac{p-q}{p+q} nx + \frac{p-q}{p+q} n \cdot \frac{n-1}{2} \cdot \frac{qn}{p+q} \frac{x^2}{a} + \&c.$$

Then, by equating the corresponding terms, we obtain these three equations:

$$a = a,$$

$$\frac{p-q}{p+q} n = 1,$$

$$\frac{n-1}{2} - \frac{qn}{p+q} = 0.$$

From which we find $\frac{p-q}{p+q} = \frac{1}{n}$ and $p:q::n+1:n-1$. So that by substituting $n+1$ and $n-1$, or any quantities proportional to them, for p and q , we shall have $\frac{n+1 \cdot N + n-1 \cdot a^n}{n-1 \cdot N + n+1 \cdot a^n} a$ for the value of the assumed quantity $\frac{pN+qa^n}{qN+pa^n} a$, which is supposed nearly equal to $a+x$, the required root of the quantity N ."

Also, in page 49—"By substituting for n , in the general theorem, severally the numbers 2, 3, 4, 5, &c. we shall obtain the following particular theorems, as adapted for the 2d, 3d, 4th, 5th, &c. roots, namely, for the

$$\text{2d or square root, } \frac{3N+a^2}{N+3a^2} a = N^{\frac{1}{2}}$$

given and assumed powers, so is the assumed root to the difference between the true and assumed roots : which difference added to or subtracted, gives the true root nearly.

That is, $\frac{n-1 \times N + n+1 \times a^n}{n+1 \times N + n-1 \times a^n} : a :: a :$

$$\frac{n+1 \cdot N + n-1 \cdot a^n}{n-1 \cdot N + n+1 \cdot a^n} a = N^{\frac{1}{n}}.$$

$$\frac{n+1 \cdot N + n-1 \cdot a^n}{n-1 \cdot N + n+1 \cdot a^n}$$

$$\text{Or, } \frac{n-1 \cdot N + n+1 \cdot a^n}{2} : N \propto a^n :: a : \frac{N \propto a^n}{\frac{n-1 \cdot N + n+1 \cdot a^n}{2}} a =$$

$$N^{\frac{1}{n}} \propto a.$$

E X A M P L E S.

1. What is the 2d root of 365 ?
2. What is the 3d root of 2.456 ?
3. What is the 4th root of 3.14159 ?
4. What is the 5th root of 5236.732 ?
5. What is the 6th root of 2.4374732 ?
6. What is the 7th root of 126.2 ?
7. What is the 8th root of 583.56 ?
8. What is the 9th root of 12.56638 ?
9. What is the 25th root of 4936.7854 ?

$$\text{3d or cube root, } \frac{4N+2a^3}{2N+4a^3} a, \text{ or } \frac{2N+a^3}{N+a^3} a = N^{\frac{2}{3}}$$

$$\text{4th root, } \frac{3N+3a^4}{3N+5a^4} a = N^{\frac{3}{4}}$$

$$\text{5th root, } \frac{6N+4a^5}{4N+6a^5} a, \text{ or } \frac{3N+2a^5}{2N+3a^5} a = N^{\frac{4}{5}}$$

$$\text{6th root, } \frac{7N+5a^6}{5N+7a^6} a = N^{\frac{5}{6}}$$

$$\text{7th root, } \frac{8N+6a^7}{6N+8a^7} a, \text{ or } \frac{4N+3a^7}{3N+4a^7} a = N^{\frac{6}{7}} \&c."$$

10. What

In Arithmetical Progression there are five quantities concerned, which may each become the subject of inquiry, and may be represented by the following letters :

The less extreme $= a$.

The greater extreme $= z$.

The common difference $= d$.

The number of terms $= n$.

The sum of the series $= s$.

Any three of the above quantities being given, the remaining two may be found ; from whence arise twenty problems, which may all be readily solved by the following

THEOREMS.

1. If from the sum of the greater extreme and common difference, the product of the common difference and number of terms be subtracted ; the remainder will be equal to the less extreme. $z + d - nd = a$.

2. If to the product of the common difference and number of terms, the less extreme be added, and from that sum the common difference be subtracted, the remainder will be equal to the greater extreme. $nd + a - d = z$.

3. If the difference of the extremes be divided by the number of terms minus one, the quotient will be equal to the common difference.

$$\frac{z - a}{n - 1} = d.$$

4. If the difference of the extremes be divided by the common difference, and the quotient increased by one, the result

$$\text{will be equal to the number of terms. } \frac{z - a}{d} + 1 = n.$$

5. If half the sum of the extremes be multiplied by the number of terms, the product will be equal to the sum of the

$$\text{series. } \frac{z + a}{2} \cdot n = s.$$

6. If twice the sum of the series be divided by the number of terms, and the greater extreme subtracted from the quotient,

$$\text{the remainder will be equal to the less extreme. } \frac{2s}{n} - z = a.$$

7. If

7. If twice the sum of the series be divided by the number of terms, and the less extreme subtracted from the quotient, the remainder will be equal to the greater extreme. $\frac{2s}{n} - a = z$.

8. If the difference of the squares of the extremes be divided by twice the sum of the series made less by the sum of the extremes, the quotient will be equal to the common difference.

$$\frac{z^2 - a^2}{2s - z - a} = d.$$

9. If twice the sum of the series be divided by the sum of the extremes, the quotient will be equal to the number of terms.

$$\frac{2s}{z + a} = n.$$

10. If the difference of the squares of the extremes be divided by twice the common difference, and the quotient increased by half the sum of the extremes, the result will be equal

$$\text{to the sum of the series. } \frac{z^2 - a^2}{2d} + \frac{z + a}{2} = s.$$

11. If to the product of the greater extreme and common difference, the square of the greater extreme, and one fourth of the square of the common difference, be added; and from that sum, twice the product of the sum of the series and common difference be taken away, the square root of the remainder added to or subtracted from one half the common difference, will give a result equal to the less extreme.

$$a = \frac{d}{2} \pm \sqrt{z^2 + dz + \frac{d^2}{4} - 2sd}^{\frac{1}{2}}.$$

12. If to twice the product of the sum of the series and common difference, the square of the less extreme, and one fourth of the square of the common difference be added; and from that sum, the product of the less extreme and common difference be taken away, the square root of the remainder, made less by one half the common difference, will give a result equal

$$\text{to the greater extreme. } z = 2sd + a^2 + \frac{d^2}{4} - ad^{\frac{1}{2}} - \frac{d}{2}.$$

13. If from twice the product of the number of terms and greater extreme, twice the sum of the series be subtracted, and the remainder divided by the difference between the number of terms

terms and its square, the quotient will be equal to the common

difference. $\frac{2nz - 2s}{n^2 - n} = d.$

14. If to twice the greater extreme the common difference be added, and from the square of that sum eight times the product of the common difference and sum of the series be subtracted, and the square root of the remainder added to or subtracted from the sum of twice the greater extreme and common difference, and this last result divided by twice the common difference, the quotient will be equal to the number of

terms. $n = \frac{2z + d \pm \sqrt{2z + d^2 - 8ds}}{2d}.$

15. If to twice the greater extreme the common difference be added, and from that sum the product of the number of terms and common difference be subtracted, and the remainder multiplied by the number of terms; one half the product will be equal to the sum of the series. $\frac{2z + d - nd}{2} \cdot n = s.$

16. If to twice the sum of the series the product of the number of terms and common difference be added, and from that sum the product of the square of the number of terms and common difference be subtracted, and the remainder divided by twice the number of terms, the quotient will be equal to the less extreme. $\frac{2s + nd - n^2d}{2n} = a.$

17. If to twice the sum of the series the product of the common difference and square of the number of terms be added, and from that sum the product of the number of terms and common difference be subtracted, and the remainder divided by twice the number of terms, the quotient will be equal to the greater extreme. $\frac{2s + n^2d - nd}{2n} = z.$

18. If from twice the sum of the series, twice the product of the number of terms and less extreme be subtracted, and the remainder divided by the difference between the number of terms and its square, the quotient will be equal to the common difference. $\frac{2s - 2na}{n^2 - n} = d.$

19. If from twice the less extreme the common difference be subtracted, and to the square of the remainder, eight times the product

product of the common difference and sum of the series be added, and the square root of the sum added to or subtracted from the difference between the less extreme and common difference, and this last result divided by twice the common difference, the quotient will be equal to the number of terms:

$$n = \frac{2a - d \pm \sqrt{2a - d^2 + 8ds}}{2d}$$

20. If to twice the less extreme the product of the number of terms and common difference be added, and from that sum the common difference be subtracted, and the remainder multiplied by the number of terms; one half the product will be equal to the sum of the series.

$$\frac{2a + nd - d \cdot n}{2} = s.$$

EXAMPLES.

1. Let $a = 2$, $z = 299$, $d = 3$, $n = 100$, and $s = 15050$; required any two from the other three?
2. Given $a = 3$, $z = 1599$, $d = 4$; required n and s ?
3. Given $a = 3$, $n = 400$, $s = 320400$; required z and d ?
4. Given $d = 2$, $n = 500$, $z = 1001$; required a and s ?
5. Given $a = 2$, $d = 5$, $s = 899700$; required n and z ?
6. Given $a = 5$, $n = 800$, $z = 9596$; required d and s ?

GEOMETRICAL PROGRESSION*.

A GEOMETRICAL PROGRESSION is a series of equimultiples, or rank of numbers, the succeeding terms of which are either all the same multiple, or all the same submultiple of their adjacent preceding terms.

The quantities concerned in geometrical progression are the same

* The fundamental properties of Arithmetical and Geometrical Progressions are so related, that whatever holds true in regard of addition, subtraction, multiplication, and division of terms in the former, holds equally true in regard of multiplication, division, involution, and evolution of like terms in the latter, as will easily appear by comparing the values of their respective terms:

same in number, and may be represented by the same letters, as in arithmetical progression :

The less extreme $= a$.

The greater extreme $= z$.

The common ratio $= d$.

The number of terms $= n$.

The sum of the series $= s$.

Any three of the above quantities being given, the remaining two may be found ; from whence, in this as well as in arith.

$$\left. \begin{array}{l} \text{Arithmetical Progression.} \\ a = z - d \cdot n - 1 \\ z = a + d \cdot n - 1 \\ d = \frac{z - a}{n - 1} \\ n = \frac{z - a}{d} + 1 \\ s = na + d \cdot \frac{n^2 - n}{2} \end{array} \right\}$$

$$\left. \begin{array}{l} \text{Geometrical Progression.} \\ a = \frac{z}{dn - 1} \\ z = ad^{n-1} \\ d = \frac{\frac{z}{a^{n-1}} - 1}{1} \\ n = \frac{L.z - L.a}{L.d} + 1 \\ p = a^n d^{\frac{n^2 - n}{2}} \end{array} \right\}$$

It is plain, from the nature of the series, that the greater extreme is equal to the less extreme multiplied by the common ratio so often as there are units in $n - 1$, from which consideration the above values of a , z , d , and n are easily derived. It is also evident, that

p , the product of all the terms, is $= a^n d^{\frac{n^2 - n}{2}}$: for the index of a will be equal to n the number of terms, and the index of d will be equal to the sum of all its indices in the several terms, but the sum of these indices is $= \frac{n^2 - n}{2}$; therefore $p = a^n d^{\frac{n^2 - n}{2}}$.

An equation containing a , z , d , and s , may be obtained from a well known property of continual proportionals ; i. e. as any one of the antecedents is to its consequent, so is the sum of all the antecedents to the sum of all the consequents, and therefore $a : ad :: s - z : s - a$; which analogy affords the following equations, viz.

$$a = dz + s - ds, z = \frac{ds + a - s}{d}, d = \frac{s - a}{s - z}, s = \frac{dz - a}{d - 1}.$$

From equating these values of a , z , and d , with the former, all the other values mentioned may be obtained.

metical

metical progression, twenty problems arise; all which may be solved by the following

THEOREMS.

1. If the common ratio be involved until the index of the power be equal to $n-1$, and the result made a divisor to the greater extreme, the quotient will be equal to the less; and if the less extreme be multiplied by the said power, the product will be equal to the greater. $a = \frac{z}{d^{n-1}}$, and $z = ad^{n-1}$.

2. If the difference between the sum of the series and less extreme, be divided by the difference between the sum of the series and greater extreme, the quotient will be equal to the common ratio. $\frac{s-a}{s-z} = d$.

3. If the difference of the logarithms of the two extremes be divided by the logarithm of the common ratio, and the quotient increased by one, the result will be equal to the number of terms. $\frac{L.z - L.a}{L.d} + 1 = n$.

4. If from the product of the greater extreme and common ratio, the less extreme be taken away, and the remainder divided by the common ratio minus one, the quotient will be equal to the sum of the series. $\frac{dz-a}{d-1} = S$.

5. If the common ratio be involved until the index of the power be equal to n , and the result made less by one, the remainder will be to the sum of the series, as the common ratio minus one to the less extreme. $\frac{ds-s}{d^n-1} = a$.

6. If to the product of the sum of the series and common ratio minus one, the less extreme be added, and the sum divided by the common ratio, the quotient will be equal to the greater extreme. $\frac{ds-s+a}{d} = z$.

7. If the greater extreme be divided by the less, the $\frac{1}{n}-1$ th root of the quotient will be equal to the common ratio. $\frac{\sqrt[n]{z}}{\sqrt[n]{a}} = d$.

8. If

8. If to the product of the sum of the series and common ratio minus one, the less extreme be added, and from the logarithm of the sum the logarithm of the less extreme be taken away, and the remainder divided by the logarithm of the common ratio; the quotient will be equal to the number of

terms.
$$\frac{L \cdot s \cdot d - s + a - L \cdot a}{L \cdot d} = n.$$

9. If the common ratio be involved until the index of the power be equal to n , and the result made less by one, the common ratio minus one, will be to the remainder, as the less

extreme to the sum of the series.
$$\frac{d^n - 1}{d - 1} \cdot a = s.$$

10. If to the product of the greater extreme and common ratio the sum of the series be added, and from that result the product of the common ratio and sum of the series be taken away, the remainder will be equal to the less extreme.
$$dx + s - sd = a.$$

11. If the common ratio be involved until the index of the power be equal to $n - 1$, and that power multiplied by the sum of the series, and the product reserved: Then, if one be subtracted from the n th power of the common ratio, the remainder will be to the reserved product, as the common ratio

minus one to the greater extreme.
$$\frac{s d^n - s d^{n-1}}{d^n - 1} = z.$$

12. If from the n th power of the common ratio, the $n - 1$ th power be taken away, the remainder will be to the n th power minus one, as the greater extreme to the sum of the series.

$$\frac{z d^n - z}{d^n - d^{n-1}} = s.$$

13. If the difference of the $\frac{n}{I}$ th powers of the extremes be divided by the difference of their $\frac{n}{I} - 1$ th roots, the quotient

will be equal to the sum of the series.
$$\frac{z^{\frac{n}{I}} - a^{\frac{n}{I}}}{\frac{1}{z^{\frac{n}{I}-1}} - \frac{1}{a^{\frac{n}{I}-1}}} = s.$$

N. B. The remaining value of each extreme may be found from the last equation; which also affords another value of n ; and the remaining value of n , in terms of s , z , and d , may be found from equa. penult.; which equation also affords another value of d ; and the remaining value of d in terms of s , a , and n , may be obtained from the equation adjoined to theorem 9th.

EXAMPLES.

1. Given $a = 2$, $d = 3$, and $z = 564859072962$; required n and s ?
2. Given $a = 2$, $n = 25$, and $z = 564859072962$; required d and s ?
3. Given $a = 2$, $z = 564859072962$, and $s = 847288609442$; required d and n ?
4. Given $a = 2$, $d = 3$, and $n = 25$; required z and s ?
5. Given $a = 2$, $d = 3$, and $s = 847288609442$; required z and n ?
6. Given $a = 2$, $n = 25$, and $s = 847288609442$; required z and d ?
7. Given $d = 3$, $n = 25$, and $z = 564859072962$; required a and s ?
8. Given $d = 3$, $z = 564859072962$, and $s = 847288609442$; required a and n ?
9. Given $n = 25$, $z = 564859072962$, and $s = 847288609442$; required a and d ?
10. Given $n = 25$, $d = 3$, and $s = 847288609442$; required a and z ?
11. A gentleman asking the price of a fine horse, was told by the owner, that if he would give one small pin, in value $\frac{1}{4}$ of a penny, for the first nail in his shoes, 3 for the second, 9 for the third, and so on to the last, or 32d. he should not only have the horse, but also a saddle worth 100*l.*; required the price of the horse?
Ans. The horse and saddle cost 60319667505*l.* 17*s.* 1*d.*; and the horse 60319667505*l.* 17*s.* 1*d.*
12. One Sessa, an Indian, having first discovered the game of Chess, shewed it to Shehram his prince, who was so delighted with the invention, that he desired him to ask what he would as a reward for his ingenuity: upon which Sessa requested, that he might be allowed one grain of wheat for the first square on the chess-board,

N

two

two for the second, and so on, doubling continually, to the last, or 64th. The king was offended at his asking so trifling a reward; but soon found it impossible to grant the request: I demand the quantity of wheat, and the number of acres which would produce it at 10 quarters per acre? *Ans.* 1844674407370955 1615 grains, of which if 491520 = one bushel, we shall have 4691249611844.25 quarters, and the required number of acres = 469124961184.425.

SIMPLE INTEREST.

SIMPLE INTEREST is an allowance made by the borrower of any sum of money to the lender thereof, according to a certain rate per centum per annum; which is either dependent on the will of the parties, or limited by law.

In Simple Interest there are five quantities concerned, which may each become the subject of inquiry.

1. The principal, or sum of money lent = P .
2. The time of continuance at interest = T .
3. The ratio, or rate per cent. affixed = R .
4. The interest, or sum gained by the loan = I .
5. The amount, or sum of principal and interest = A .

Their several values may be found in terms of each other by the following

RULES*.

- I. Divide the amount by the product of the rate and time, increased by one; or divide the interest by the product of the time and rate; the quotient will be the principal.

$$\frac{A}{TR + 1} = \frac{I}{TR} = P.$$

II. Divide

* That the interest should be as the principal, rate, and time conjointly, is evident; as they are all variable: for let m be any given interest per centum per annum, or the interest of 100l. for one year; then as $100 : 1 :: m : r$ = the interest of one pound for one year, or $\frac{m}{100} = \frac{r}{1}$ the ratio in lowest terms; again $1 : t ::$

$r : tr$

II. Divide the interest, or difference of the amount and principal, by the product of the principal and rate; the quotient will be the time of continuance. $\frac{A-P}{P R} = \frac{I}{P R} = T.$

III. Divide the interest, or difference of the amount and principal, by the product of the principal and time; the quotient will be the rate affixed. $\frac{A-P}{P T} = \frac{I}{P T} = R.$

IV. The sum of principal and interest, or the continual product of principal, rate, and time, increased by the principal, is equal to the amount. $PRT + P = P + I = A.$

V. The continual product of the principal, rate, and time, or the difference of amount and principal, is equal to the interest, or sum gained by the loan. $PRT = A - P = I.$

$r : tr =$ the interest of one pound for the number of years expressed by t ; and likewise, $1 : P :: tr : P tr = i$ the interest of p pounds for t years.

That the amount should be = the sum of principal and interest is self-evident; and therefore we have two equations $P tr = i$, and $P tr + P = a$, from whence the others are easily derived. The three following columns exhibit the value of each when t is taken in years, months and days respectively.

$Ptr = a - P = i.$	$\frac{Ptr}{12} = a - P = i.$	$\frac{Ptr}{365} = a - P = i.$
$Ptr + P = P + i = a.$	$\frac{Ptr + 12P}{12} = P + i = a.$	$\frac{Ptr + 365P}{365} = P + i = a.$
$\frac{a - P}{Pr} = \frac{i}{Pr} = t.$	$\frac{a - P \times 12}{Pr} = \frac{12i}{Pr} = t.$	$\frac{a - P \times 365}{Pr} = \frac{365i}{Pr} = t.$
$\frac{a - P}{Pt} = \frac{i}{Pt} = r.$	$\frac{a - P \times 12}{Pt} = \frac{12i}{Pt} = r.$	$\frac{a - P \times 365}{Pt} = \frac{365i}{Pt} = r.$
$\frac{a}{tr + 1} = \frac{i}{tr} = P.$	$\frac{12a}{tr + 12} = \frac{12i}{tr} = P.$	$\frac{365a}{tr + 365} = \frac{365i}{tr} = P.$

A TABLE OF RATIOS.

1 = .01	$3\frac{1}{2}$ = .035	6 = .06	$8\frac{1}{2}$ = .085
$1\frac{1}{2}$ = .015	4 = .04	$6\frac{1}{2}$ = .065	9 = .09
2 = .02	$4\frac{1}{2}$ = .045	7 = .07	$9\frac{1}{2}$ = .095
$2\frac{1}{2}$ = .025	5 = .05	$7\frac{1}{2}$ = .075	10 = .1
3 = .03	$5\frac{1}{2}$ = .055	8 = .08	$10\frac{1}{2}$ = .105

The interest of 1 pound for 1 year.

The Interest of one Pound for one Day.

1 = .00002739726	$4\frac{1}{2}$ = .00012328767	8 = .00021917808
$1\frac{1}{2}$ = .00004109589	5 = .00013698630	$8\frac{1}{2}$ = .00023287671
2 = .00005479452	$5\frac{1}{2}$ = .00015068493	9 = .00024657534
$2\frac{1}{2}$ = .00006849315	6 = .00016438356	$9\frac{1}{2}$ = .00026027397
3 = .00008219178	$6\frac{1}{2}$ = .00017808219	10 = .00027397260
$3\frac{1}{2}$ = .00009589041	7 = .00019178082	$10\frac{1}{2}$ = .00028767123
4 = .00010958904	$7\frac{1}{2}$ = .00020547945	11 = .00030136986

A T A B L E,

Shewing the number of Days from any Day of one Month to the same Day of any other Month.

To	From any one Day of											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
Jan.	365	334	306	275	245	214	184	153	122	92	61	31
Feb.	31	365	337	306	276	245	215	184	153	122	92	62
Mar.	59	28	365	334	304	273	243	212	181	151	120	90
Apr.	90	59	31	365	335	304	274	243	212	182	151	121
May	120	89	61	30	365	334	304	273	242	212	181	151
June	151	120	92	61	31	365	335	304	273	243	212	182
July	181	150	122	91	61	30	365	334	303	273	242	212
Aug.	212	181	153	122	92	61	31	365	334	304	273	243
Sept.	243	212	184	153	122	92	62	31	365	335	304	274
Oct.	273	242	214	183	153	122	92	61	30	365	334	304
Nov.	304	273	245	214	184	153	123	92	61	31	365	335
Dec.	334	303	275	244	214	183	153	122	91	61	30	365

EXAMPLES.

1. What is the interest of 945 pounds 10 shillings for 27 years, at 5 per cent. per annum? *Ans.* 1276l. 8s. 6d.
2. What is the interest of 796 pounds 15 shillings for 30 years, at $4\frac{1}{2}$ per cent. per annum? *Ans.* 1075l. 12s. 3d.
3. What is the interest of 886 pounds 18 shillings and 9 pence, for $16\frac{1}{4}$ years, at 5 per cent. per annum? *Ans.* 576l. 10s. 2d. 1q.
4. What is the interest of 8560 pounds 10 shillings for 73 days, at 5 per cent per annum? *Ans.* 85l. 12s. 1.2d.
5. What is the interest of 668 pounds 15 shillings for 273.75 days, at 4 per cent. per annum? *Ans.* 20l. 0s. 1d. 1q.
6. What is the amount of 945 pounds 10 shillings for 27 years, at 5 per cent. per annum? *Ans.* 1771l. 18s. 6d.
7. What is the amount of 796 pounds 15 shillings for 30 years, at $4\frac{1}{2}$ per cent. per annum? *Ans.* 1872l. 7s. 3d.
8. What is the amount of 886l. 18s. and 9d. for $16\frac{1}{4}$ years, at 4 per cent. per annum? *Ans.* 1463l. 8s. 11d. 1q.
9. What is the amount of 8560l. 10s. for 73 days, at 5 per cent. per annum? *Ans.* 8646l. 2s. 1.2d.
10. What is the amount of 668l. 15s. for 273.75 days, at 4 per cent. per annum? *Ans.* 688l. 15s. 1d. 1q.
11. What principal will amount to 1771l. 18s. 6d. in 27 years, at 5 per cent. per annum? *Ans.* 945l. 10s.
12. What principal will amount to 1872l. 7s. 3d. in 30 years, at $4\frac{1}{2}$ per cent. per annum? *Ans.* 796l. 15s.
13. What principal will amount to 1463l. 8s. 11d. 1q. in $16\frac{1}{4}$ years, at 4 per cent. per annum? *Ans.* 886l. 18s. 9d.
14. What principal will amount to 8646l. 2s. 1.2d. in 73 days, at 5 per cent. per annum? *Ans.* 8560l. 10s.
15. What principal will amount to 688l. 15s. 1d. 1q. in 273.75 days, at 4 per cent. per annum. *Ans.* 668l. 15s.
16. In what time will 945l. 10s. amount to 1771l. 18s. 6d. at 5 per cent. per annum? *Ans.* 27 years.
17. In what time will 796l. 15s. amount to 1872l. 7s. 3d. at $4\frac{1}{2}$ per cent. per annum? *Ans.* 30 years.
18. In what time will 886l. 18s. 9d. amount to 1463l. 8s. 11d. 1q. at 4 per cent. per annum? *Ans.* $16\frac{1}{4}$ years.
19. In what time will 8560l. 10s. amount to 8646l. 2s. 1.2d. at 5 per cent. per annum? *Ans.* 73 days.

20. In

20. In what time will 668l. 15s. amount to 688l. 15s. 1d. 1q. at 4 per cent. per annum? *Ans.* 273.75 days.
21. At what rate per cent. will 945l. 10s. amount to 1771l. 18s. 6d. in 27 years? *Ans.* 5 per cent.
22. At what rate per cent. will 796l. 15s. amount to 1872l. 7s. 3d. in 30 years? *Ans.* $4\frac{1}{2}$ per cent.
23. At what rate per cent. will 886l. 18s. 9d. amount to 1463l. 8s. 11d. 1q. in $16\frac{1}{4}$ years? *Ans.* 4 per cent.
24. At what rate per cent. will 856ol. 10s. amount to 8646l. 2s. 1.2d. in 73 days? *Ans.* 5 per cent.
25. At what rate per cent. will 668l. 15s. amount to 688l. 15s. 1d. 1q. in 273.75 days? *Ans.* 4 per cent.

COMPOUND INTEREST.

COMPOUND INTEREST is that arising from the amount considered as new principal, after the time of each succeeding payment.

The quantities concerned in Compound Interest are the same in number, and may be represented by the same letters, as in Simple Interest, with this difference, that R represents the amount of one pound at the time of the first payment, in place of its interest.

The values of the several quantities may be found in terms of each other by the following

RULES*.

- I. Find the amount of one pound at the time of the first payment; involve this amount to a power, the index of which

* Demon. Since one pound at the time of the first payment becomes R , it will be as $1 : R :: R : R^2$, the amount of one pound at the time of the second payment; and as $1 : R :: R^2 : R^3$, the amount of one pound at the time of the third payment: whence it appears that R^T , or R raised to that power, the index of which is equal to the number of payments, will be the amount of one pound at the time of T payments; but as $1 : R^T :: P : P \times R^T = A$, the amount of P in the time T ; and if this amount be made less by the principal, it is plain that the remainder will be equal to the accumulated interest, (i. e.) $P \times R^T - P = I$: from which equations the others are easily derived. Q. E. D.

is equal to the number of payments; multiply this power by the given principal; and the product will be equal to the amount. $P \times R^T = A$, $\left\{ \begin{array}{l} P \times R^T - P = I. \end{array} \right.$

II. From the amount subtract the principal, and the remainder will be the accumulated interest.

III. Find the amount of one pound at the time of the first payment; involve this amount to a power, the index of which is equal to the number of payments; divide the amount by this power, and the quotient will be equal to the principal.

$$P = \frac{A}{R^T}$$

IV. Divide the amount by the principal; extract the root from the quotient, the index of which is the reciprocal of the time, and the result will be equal to the amount of one pound at the time of the first payment, from which if one be subtracted, the remainder will be the rate per cent.

$$\sqrt[T]{\frac{A}{P}} = R, \text{ and } R - 1 = r, \text{ the rate as in Simple Interest.}$$

The time is found by the solution of an exponential equation; and hence the following logarithmic solution.

I. Multiply the logarithm of R by T ; to which add the logarithm of P , and the sum will be equal to the logarithm of A .

$$A \cdot T \times L \cdot R + L \cdot P = L \cdot A.$$

II. From the logarithm of A subtract the product of T , multiplied by the logarithm of R ; and the difference will be equal to the logarithm of P . $L \cdot A - T \times L \cdot R = L \cdot P$.

III. From the logarithm of A subtract the logarithm of P ; divide the difference by T , and the quotient will be equal to the logarithm of R . $\frac{L \cdot A - L \cdot P}{T} = L \cdot R$.

IV. From the logarithm of A subtract the logarithm of P ; divide the difference by the logarithm of R , and the quotient will be equal to T . $\frac{L \cdot A - L \cdot P}{L \cdot R} = T$.

EXAMPLES.

1. What is the amount of 888l. for 25 years, at 5 per cent. compound interest? *Ans.* 3007l. 2s.
2. What is the amount of 684l. 16s. for 36 years, at 6 per cent. per annum? *Ans.* 5579l. 6s. 6d.
3. What is the amount of 2998l. 14s. for 28 years, at 4 per cent. per annum? *Ans.* 8992l. 4s.
4. What is the amount of 2731l. 18s. for 34 years, at 3 per cent. per annum? *Ans.* 7463l. 6s.
5. What is the amount of 4466l. 12s. for 42 years, at 8 per cent. per annum? *Ans.* 11318l. 6s. 8d.
6. What is the interest of 888l. for 25 years, at 5 per cent. per annum? *Ans.* 2119l. 2s.
7. What is the interest of 684l. 16s. for 36 years, at 6 per cent. per annum? *Ans.* 4894l. 10s. 6d.
8. What is the interest of 2998l. 14s. for 28 years, at 4 per cent. per annum? *Ans.* 5993l. 10s.
9. What is the interest of 2731l. 18s. for 34 years, at 3 per cent. per annum? *Ans.* 4731l. 8s.
10. What is the interest of 4466l. 12s. for 42 years, at 8 per cent. per annum? *Ans.* 108714l. 14s. 8d.
11. What principal will amount to 3007l. 2s. in 25 years, at 5 per cent. per annum? *Ans.* 888l.
12. What principal will amount to 5579l. 6s. 6d. in 36 years, at 6 per cent. per annum? *Ans.* 684l. 16s.
13. What principal will amount to 8992l. 4s. in 28 years, at 4 per cent. per annum? *Ans.* 2998l. 14s.
14. What principal will amount to 7463l. 6s. in 34 years, at 3 per cent. per annum? *Ans.* 2731l. 18s.
15. What principal will amount to 11318l. 6s. 8d. in 42 years, at 8 per cent. per annum? *Ans.* 4466l. 12s.
16. In what time will 888l. amount to 3007l. 2s. at 5 per cent. per annum? *Ans.* 25 years.
17. In what time will 684l. 16s. amount to 5579l. 6s. 6d. at 6 per cent. per annum? *Ans.* 36 years.
18. In what time will 2998l. 14s. amount to 8992l. 4s. at 4 per cent. per annum? *Ans.* 28 years.
19. In what time will 2731l. 18s. amount to 7463l. 6s. at 3 per cent. per annum? *Ans.* 34 years.
20. In what time will 4466l. 12s. amount to 11318l. 6s. 8d. at 8 per cent. per annum? *Ans.* 42 years.

21. At

142 ANNUITIES AT SIMPLE INTEREST.

21. At what rate per cent. will 888l. amount to 3007l. 2s. in 25 years? *Ans.* 5 per cent.
22. At what rate per cent. will 684l. 16s. amount to 5579l. 6s. 6d. in 36 years? *Ans.* 6 per cent.
23. At what rate per cent. will 2998l. 14s. amount to 8992l. 4s. in 28 years? *Ans.* 4 per cent.
24. At what rate per cent. will 2731l. 18s. amount to 7463l. 6s. in 34 years? *Ans.* 3 per cent.
25. At what rate per cent. will 4466l. 12s. amount to 11318l. 6s. 8d. in 42 years? *Ans.* 8 per cent.

ANNUITIES AT SIMPLE INTEREST.

ANNUITIES are sums of money payable per annum for a limited time, or for ever.

Annuities are said to be in arrear, when they are retained in the hands of the debtor beyond the time of payment.

The sum of all the annuities for the time they have been forborn, together with the interest due upon each, is called the amount.

That principal, which, if put to interest at the same rate, and continued until the expiration of the annuity, would amount to the same sum, may be called the present worth of that annuity.

The present worth calculated for any time prior to the commencement of the annuity, may be called the purchase in reversion.

If u represent any annuity, P its present worth, and A, I, R, T , as before, their several values, at Simple Interest, may be found in terms of each other by the following rules *.

I. Find

* Demon. The interest due on the first year's annuity at the end of t years, will be $t - 1 \times ru$; that on the second $t - 2 \times ru$; that on the third $t - 3 \times ru$; and so on to the last, which is $t - t \times ru = 0$. From whence it is manifest, that the whole interest to be received at the expiration of t years will be truly defined by

ANNUITIES AT SIMPLE INTEREST. 143

- I. Find the sum of the series $1 + 2 + 3 + 4$, &c. to $t - 1$ terms; multiply this sum by one year's interest of the annuity, and the product will be the whole interest due at the end of t years. $\frac{t^2 - 1}{2} \times UR = I.$

by the arithmetical progression $ru + 2ru + 3ru + 4ru +$, &c. to $t - 1$ terms: but the sum of this progression is $= \frac{t^2 - 1}{2} \times$

$ru = i$, which is rule 1. And it is also manifest, that if to this interest all the annuities be added, the sum will be equal to the total amount; but the sum of all the annuities is $= tu$: therefore

$\frac{t^2 - 1}{2} \times ru + tu = a$, which is rule 2d; and from these two equations the 3d, 4th, and 5th are easily derived.

That rule 6th is strictly true, agreeable to the definition which I have given of present worth, is manifest; because, the time and rate being the same, the amounts will be as the principals. Therefore this rule, with the five preceding, are agreeable to the hypotheses. Q. E. D. The other rule marked with inverted commata, is that given by Mr. Kersey, and raised from the dead by a late author, after it had been for many years damned. If the following definition of present worth be admitted, Mr. Kersey's rule is undeniably true.

The present worth of any annuity is that sum of money which, if put to interest at a given rate, and the several annuities paid therefrom as they become due, the remainder always bearing interest as before, shall at the expiration of the annuity be totally

exhausted. The present worth in this case becomes $P = \frac{u}{1 + r} +$

$\frac{u}{1 + 2r} + \frac{u}{1 + 3r} + \frac{u}{1 + 4r} +$ &c. to $\frac{u}{1 + tr}.$ When compound

interest is allowed to the purchaser, both rules are perfectly the same; but when simple interest only is allowed, they will always differ by a given quantity. The truth or fallacy of any rule of this kind depends entirely upon its agreement, or disagreement, with the data or conditions of the proposition, equity and justice being entirely out of the question in regard to the purchase of annuities at simple interest.

144 ANNUITIES AT SIMPLE INTEREST.

II. To the former product, or value of 1, add the product of the annuity and time, and the sum will be the whole amount

at the end of T years.
$$\frac{T^2 - T}{2} \times UR + UT = A.$$

III. Multiply the sum of the series mentioned in rule 1st, by the rate per cent. to which product add the number of payments, or value of T ; make the result a divisor, and the amount a dividend; the quotient will be equal to the annuity.

$$\frac{2A}{T^2 - T \times R + 2T} = \frac{21}{T^2 - T \times R} = U.$$

IV. Multiply the sum of the series abovementioned, by the annuity, and make the product a divisor; from twice the amount subtract twice the product of time and annuity, and make the remainder a dividend; the quotient will be equal

to the rate per cent.
$$\frac{2A - 2TU}{T^2 - T \times U} = R.$$

V. Divide the number 2 by the rate per cent. subtract one from the quotient, and call the remainder N . Divide twice the amount by the product of the annuity and rate; to the quotient add one fourth of the square of N , and extract the square root of the sum; from this root subtract one half of N , and the remainder will be equal to the time of con-

tinuance.
$$\frac{2A}{UR} + \frac{N^2}{4} - \frac{N}{2} = T.$$

VI. As the amount of one pound, from the time of purchase to that of expiration, is to the amount of the annuity; so is one pound to the present worth of that annuity. $TR + 1:$

$$\frac{T^2 - T \times R + 2T}{2} \times U = A :: 1 : \frac{T^2 - T \times R + 2T}{2TR + 2} \times U = P.$$

The following is founded on a different hypothesis.

“Find the present worth of each payment by itself, discounting from the time it falls due, and the sum of all these will be the present worth required.”

EXAMPLES.

1. What is the amount of 250l. annuity forborn 7 years, at 6 per cent. simple interest? *Ans.* 2065l.

2. What is the amount of 150l. annuity forborn 9 years, at 6 per cent.? *Ans.* 1674l.

3. What

ANNUITIES IN SIMPLE INTEREST. 145

3. What is the amount of 600*l.* annuity forborn 12 years, at 5 per cent. ?
Ans. 1116*l.*
4. What is the amount of 1200*l.* annuity forborn 15 years, at 4 per cent. ?
Ans. 2808*l.*
5. What is the interest of 250*l.* annuity, forborn 7 years, at 6 per cent. ?
Ans. 315*l.*
6. What is the interest of 150*l.* annuity forborn 9 years, at 6 per cent. ?
Ans. 324*l.*
7. What is the interest of 600*l.* annuity forborn 12 years, at 5 per cent. ?
Ans. 396*l.*
8. What is the interest of 1200*l.* annuity forborn 15 years, at 4 per cent. ?
Ans. 1008*l.*
9. What annuity will amount to 2065*l.* in 7 years, at 6 per cent. ?
Ans. 250*l.*
10. What annuity will amount to 1674*l.* in 9 years, at 6 per cent. ?
Ans. 150*l.*
11. What annuity will amount to 1116*l.* in 12 years, at 5 per cent. ?
Ans. 600*l.*
12. What annuity will amount to 2888*l.* in 15 years, at 4 per cent. ?
Ans. 1200*l.*
13. At what rate per cent. will 250*l.* annuity amount to 2065*l.* in 7 years ?
Ans. 6 per cent.
14. At what rate per cent. will 150*l.* annuity amount to 1674*l.* in 9 years ?
Ans. 6 per cent.
15. At what rate per cent. will 600*l.* annuity amount to 1116*l.* in 12 years ?
Ans. 5 per cent.
16. At what rate per cent. will 1200*l.* annuity amount to 2808*l.* in 15 years ?
Ans. 4 per cent.
17. In what time will 250*l.* annuity amount to 2065*l.* at 6 per cent. ?
Ans. 7 years.
18. In what time will 150*l.* annuity amount to 1674*l.* at 6 per cent. ?
Ans. 9 years.
19. In what time will 900*l.* annuity amount to 1116*l.* at 5 per cent. ?
Ans. 12 years.
20. In what time will 1200*l.* annuity amount to 2808*l.* at 4 per cent. ?
Ans. 15 years.
21. What is the present worth of 250*l.* annuity for 7 years at 6 per cent. ?
Ans. 1454*l.* 4*s.* 6*d.*
22. What is the present worth of 150*l.* annuity for 9 years at 6 per cent. ?
Ans. 1087*l.* 0*s.* 2*d.*
23. What is the present worth of 600*l.* annuity for 12 years at 5 per cent. ?
Ans. 6937*l.* 10*s.*
24. What

246 ANNUITIES AT COMPOUND INTEREST.

24. What is the present worth of 1200l. annuity for 15 years at 4 per cent. ?

Ans. 1755sol.

ANNUITIES AT COMPOUND INTEREST.

Retaining the same letters as before, their several values may be found in terms of each other by the following rules*.

I. As

* *Demon.* The rate per cent. which is the interest of one pound for one year may be considered as an annuity, and the interest due upon one pound for any number of years, the amount of that annuity; but the rate and time remaining the same, the amounts will be as the principals: therefore, as the rate per cent. is to the annuity, so is the compound interest of one pound to the amount of

the annuity; that is $r : r + i^t - 1 :: u : \frac{r + i^t - 1}{r} \times u = a$,

or if $r = R$, then $a = \frac{r^t - 1}{r - 1} \times u$, according to rule 1st. And it

is plain that if from the amount the sum of all the annuities be subtracted the remainder will be the accumulated interest; but the

sum of all the annuities is $= tu$, hence $\frac{r^t - 1}{r - 1} \times u - tu = i$, according to rule second, Q. E. D.

The third rule is founded upon the same principle with the first; indeed the proportion is the same, only a different term required.

Rule fourth is the same with rule sixth in annuities at Simple Interest, and is equally applicable to both, and indeed to every species of regular interest whatever.

Rule fifth is the same with rule fourth in Compound Interest, the principal in the one being the same with the present worth in the other.

The rule which I have given for the time, is the same with that in Compound Interest; the other which is found independent of the present worth I have also given an expression for; but the rate per cent. cannot be found independent of the present worth, without a great deal of trouble; the equation from which its value may be derived, is $r^t - 1 + \frac{ar}{a} + \frac{a}{u} = 0$.

ANNUITIES AT COMPOUND INTEREST. 147

I. As the rate per cent. is to the annuity, so is the compound interest of one pound to the amount of that annuity.

$$R - I : R^T - I :: U : \frac{R^T - I \times U}{R - I} = A.$$

II. From the amount subtract the sum of all the annuities, and the remainder will be the accumulated interest.

$$\frac{R^T - I \times U}{R - I} - TU = I.$$

III. As the compound interest of one pound is to the amount of the annuity, so is the rate per cent. to that annuity.

$$R^T - I : A :: R - I : \frac{R - I \times A}{R^T - I} = U.$$

IV. As the amount of one pound, from the time of purchase, to that of expiration, is to the amount of the annuity, so is one pound to the present worth of that annuity.

$$R^T : \frac{R^T - I \times U}{R - I} = A :: I : \frac{A}{R^T} = \frac{I - \frac{I}{R^T} \times U}{R - I} = P.$$

V. Divide the amount of the annuity by its present worth; extract the root from the quotient, the index of which is the reciprocal of the time from purchase to expiration, and subtract one from the result, the remainder will be the rate per

cent. $\left[\frac{A}{P} \right]^{\frac{1}{T}} - I = R - I = r$. The time is found by the solution of an exponential equation, to which logarithms are best adapted.

VI. From the logarithm of the amount, subtract the logarithm of the present worth, divide the difference by the lo-

If t be infinite, or the annuity to continue for ever, then the amount of one pound will become $\frac{r - I}{1 - r}$, and that of the annuity

$\frac{u}{1 - r}$; but these are to one another as $r - I : u$; therefore per

cent. $r - I : u :: 1 : \frac{u}{r - I}$ = the present worth of any freehold, or real estate.

148 ANNUITIES AT COMPOUND INTEREST.

garithm of R , and the quotient will be equal to the time.

$$\frac{L.A - L.P}{L.R} = \frac{L.AR - A + U}{R.R} - L.U = T.$$

EXAMPLES.

1. What is the amount of 600l. annuity forborn 25 years, at 6 per cent. common interest? *Ans.* 32918l. 18s. 2.4d.
2. What is the amount of 500l. annuity forborn 36 years, at 5 per cent.? *Ans.* 47918l. 3s.
3. What is the amount of 800l. annuity forborn 42 years, at 4 per cent.? *Ans.* 83855l. 13s. 7.2d.
4. What is the amount of 300l. annuity forborn 50 years, at 3 per cent.? *Ans.* 33839l. 1.2s.
5. What is the interest of 600l. annuity forborn 25 years, at 6 per cent.? *Ans.* 32318l. 18s. 2.4q.
6. What is the interest of 500l. annuity forborn 36 years, at 5 per cent.? *Ans.* 47418l. 3s.
7. What is the interest of 800l. annuity forborn 42 years, at 4 per cent.? *Ans.* 83055l. 13s. 7.2d.
8. What is the interest of 300l. annuity forborn 50 years, at 3 per cent.? *Ans.* 33539l. 1.2s.
9. What annuity will amount to 32918l. 18s. 2.4d. in 25 years, at 6 per cent.? *Ans.* 600l.
10. What annuity will amount to 47918l. 3s. in 36 years, at 5 per cent.? *Ans.* 500l.
11. What annuity will amount to 83855l. 13s. 7.2d. in 42 years, at 4 per cent.? *Ans.* 800l.
12. What annuity will amount to 33839l. 1.2s. in 50 years, at 3 per cent.? *Ans.* 300l.
13. At what rate per cent. will 600l. annuity amount to 32918l. 18s. 2.4d. in 25 years? *Ans.* 6 per cent.
14. At what rate per cent. will 500l. amount to 47918l. 3s. in 36 years? *Ans.* 5 per cent.
15. At what rate per cent. will 800l. annuity amount to 83855l. 13s. 7.2d. in 42 years? *Ans.* 4 per cent.
16. At what rate per cent. will 300l. annuity amount to 33839l. 1.2s. in 50 years? *Ans.* 3 per cent.
17. In what time will 600l. annuity amount to 32918l. 18s. 2.4d. at 6 per cent.? *Ans.* 25 years.
18. In what time will 500l. annuity amount to 47918l. 3s. at 5 per cent.? *Ans.* 36 years.

19. In

19. In what time will 800l. annuity amount to 83855l. 13s. 7.2d. at 4 per cent. ? *Ans.* 42 years.
20. In what time will 300l. annuity amount to 33839l. 1.2s. at 3 per cent. ? *Ans.* 50 years.
21. What is the present worth of 600l. annuity for 25 years, at 6 per cent. ? *Ans.* 7670l. 0s. 6d.
22. What is the present worth of 500l. annuity for 36 years, at 5 per cent. ? *Ans.* 8273l. 8s. 6.12d.
23. What is the present worth of 800l. annuity for 42 years, at 4 per cent. ? *Ans.* 16148l. 10s. 348d.
24. What is the present worth of 300l. annuity for 50 years, at 3 per cent. ? *Ans.* 7719l. 1s. 3.888d.
25. What is the purchase of a freehold estate of 500l. per an. granting 5 per cent to the purchaser ? *Ans.* 10000l.

REBATE, OR DISCOUNT.

REBATE, or DISCOUNT, is the difference between a sum of money due at a certain time to come, and its present worth; or, it is the interest of the present worth during the time for which the debt was purchased.

The present worth of any debt due some time hence, is a sum of money, which, if put to interest, would in the time, and at the rate for which the discount is to be made, amount to the debt then due: from whence it appears, that present worth and discount are the same with principal and interest already explained. The following rule may serve to discover both.

RULE *.

As the amount of one pound for the given time,
and at the given rate :

Is to the given sum or debt :

So is one pound, or the interest of one pound for
the given time, and at the given rate :

O 3

To

* That the above rule for determining present worth and discount,

150 REBATE, OR DISCOUNT.

To the present worth, or the discount of the given sum or debt respectively.

count, extends to Simple and Compound Interest, is evident from what has already been said on these subjects. The following table contains proper expressions for both.

The present worth of any sum S at Simple Interest.

Rate per cent.	For t Years.	For t Months.	For t Days.
At r per cent.	$\frac{S}{rt + 1}$	$\frac{12 S}{rt + 12}$	$\frac{365 S}{rt + 365}$

The discount of any sum S at Simple Interest.

Rate per cent.	For t years.	For t Months.	For t Days.
At r per cent.	$\frac{str}{tr + 1}$	$\frac{str}{tr + 12}$	$\frac{str}{tr + 365}$

The present worth of any sum S at Compound Interest.

$$Asrt : s :: 1 : \frac{s}{r^t} = P$$

The discount of any sum S at Compound Interest.

$$Asrt : s :: rt - 1 : \frac{s \times rt - 1}{rt} = P \quad rt - P$$

EXAMPLES,

EXAMPLES.

1. What is the present worth of 1771l. 18s. 6d, due 27 years hence discounted at 5 per cent. ? *Ans.* 945l. 10s.
2. What is the present worth of 1872l. 7s. 3d. due 30 years hence discount at $4\frac{1}{2}$ per cent. ? *Ans.* 796l. 15s.
3. What is the present worth of 1463l. 8s. 11d. 1q. due $16\frac{1}{4}$ years hence, discount at 4 per cent. ? *Ans.* 886l. 18s. 9d.
4. What is the present worth of 8646l. 2s. 1.2d. due 73 days hence, discount at 5 per cent. ? *Ans.* 8560l. 10s.
5. What is the present worth of 688l. 15s. 1d. 1q. due 273.75 days hence, discount at 4 per cent. ? *Ans.* 668l. 15s.
6. What is the discount of 1771l. 18s. 6d. due 27 years hence, at 5 per cent. per an. ? *Ans.* 1276l. 8s. 6d.
7. What is the discount of 1872l. 7s. 3d. due 30 years hence, at $4\frac{1}{2}$ per cent. per annum ? *Ans.* 1075l. 12s. 3d.
8. What is the discount of 1463l. 8s. 11d. 1q. due $16\frac{1}{4}$ years hence, at 4 per cent. per annum ? *Ans.* 576l. 10s. 2d. 12.
9. What is the discount of 8646l. 2s. 1.2d. due 73 days hence, at 5 per cent. per annum ? *Ans.* 85l. 12s. 1.2d.
10. What is the discount of 688l. 15s. 1d. 1q. due 273.75 days hence, at 4 per cent. per an. ? *Ans.* 20l. 0s. 1d. 1q.

EQUATION OF PAYMENTS.

EQUATION OF PAYMENTS is the mode of finding a time, to pay at once, several debts due at different times, so that neither party shall sustain loss.

The equated time, according to different hypotheses, may be found by the following

RULES*.

- I. Multiply each payment by the time at which it becomes due

* Rule first is derived from the demonstration of the following theorem.

If any number of given sums be paid at given times, and from these

152 REBATE, OR DISCOUNT.

due; divide the sum of the products by the sum of the payments, and the quotient will be the equated time required.

The following gives the equated time for two payments.

II. Divide the sums of the two payments by the product of the

these times forward bear simple interest at a given rate; a certain time may be found, at which if one sum, equal to all the former, be put to interest at the same rate, the sum last mentioned shall, at a given period, have an amount equal to all the former.

Demon. Let the given sums be a, b, c, d , and their times of payment m, n, u, v respectively; also let x be the equated time, and v the period at which the equality of amounts shall take place.

Then $ar \times v - m + br \times v - n + cr \times v - u + dr \times v - v + a + b + c + d = a + b + c + d \times r \times v - x + a + b + c + d$;
which reduced gives $x = \frac{am + bn + cu + dv}{a + b + c + d}$ Q. E. D.

Rule second is founded upon the following hypothesis.

That the interest of the first payment shall at the time required be equal to the discount of the second,

Demon. Let a and b be the two debts, t the time between the payments, and x the equated time. Then per hypoth. $arx = \frac{btr - brx}{1 + tr - rx}$; which equation reduced gives $x^2 - x \frac{a + b}{ar} + t =$

$-\frac{bt}{ar}$, and by substitution, this last equation becomes $x^2 - mx = n$; therefore $x = m \pm \sqrt{m^2 - n}$. Q. E. D.

If in the abovementioned theorem the word compound be written in place of simple; rule third may then be derived from its demonstration, in the following manner.

Demon. Let the several debts, times of payment, equated time, and period of equal amounts, be represented by the same letters as above. The known principles of Compound Interest, will afford the following equation. $a \times r^{v-m} + b \times r^{v-n} + c$

$\times r^{v-u} + d \times r^{v-v} = a + b + c + d \times r^x$; from which

equation the following is easily derived, $x = \frac{L. a \times r^{v-m} + b \times r^{v-n} + c \times r^{v-u} + d \times r^{v-v} - L. a + b + c + d.}{L. r}$

L. r

Q. E. D.

COMPOUND PROGRESSIONS. 153

the rate per cent. and first payment; to the quotient add the time between the payments, and call the sum $2m$.

Multiply the time between the payments by the second, and divide the product by that of the rate per cent. and first payment, and call the quotient n .

From the square of m subtract n , and extract the square root of the remainder.

The sum or difference of m and this root, will be the equated time required.

The following rule gives the true equated time for any number of payments at Compound Interest.

III. Find the amount of all the debts at the time of the last payment.

From the logarithm of this amount, subtract the logarithm of the sum of all the debts, and divide the remainder by the logarithm of the rate plus one; the quotient thus found will be the equated time required.

EXAMPLES.

1. There is now due 100l. two years hence 200l. four years hence 400l. eight years hence 800l. and sixteen years hence 1600l.; I demand the equated time of payment, granting 5 per cent. per annum simple interest?
2. There is due three years hence 900l. nine years hence 2700l. twenty-seven years hence 8100l. and eighty-one years hence an estate worth 24300l. when may the whole be discharged, granting 6 per cent. per annum, compound interest?
3. There is an annuity of 600l. which will commence five years hence, and continue twenty-five years; when may the whole be paid at once, granting 6 per cent. per annum compound interest?
4. There is an annuity of 500l. which will commence six years hence, and continue thirty-six years; when may the whole be paid at once, granting 5 per cent. per annum compound interest?

COMPOUND PROGRESSIONS.

COMPOUND PROGRESSIONS are those which arise from the combination of several simple series, the varieties of which are infinite

infinite; but the most common and useful are those series of powers, or products, the roots or component factors of which are in arithmetical progression, also figurate and polygonal numbers, with their reciprocals; their powers, and products by the respective terms of other series, &c.

The sums of a few of those kinds of progressions may be found by help of the following

THEOREMS.

I. The sum of n terms of a series of squares, the sides or roots of which are in arithmetical progression, first term and com-

mon difference unity, is $= \frac{n}{1} \cdot \frac{n+1}{2} \cdot \frac{2n+1}{3} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$

II. The sum of n terms of a series of cubes is $= \frac{n^2 \cdot n+1}{4}$

$$= \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$$

III. The sum of n terms of a series biquadrates is $= \frac{n^5}{5} + \frac{n^4}{2} +$

$$\frac{n^3}{3} - \frac{n}{30}$$

IV. The sum of n terms of a series of sursolidis is $= \frac{n^6}{6} + \frac{n^5}{2} + 5$

$$\frac{n^4}{12} - \frac{n^2}{12}$$

V. The sum of n terms of a series of square cubes $= \frac{n^7}{7}$

$$+ \frac{n^6}{2} + \frac{n^5}{2} - \frac{n^3}{6} + \frac{n}{42}$$

VI. The sum of n terms of a series of squares, the sides or roots of which are in arithmetical progression, first term

$m + a$, and common difference a , is $= \frac{n \cdot m + n^2}{2} + \frac{n \cdot m a + n^2 + n \cdot 2n + 1 \cdot a^2}{6}$

VII. The sum of n terms of a series of cubes, first term and common

common difference as above, is $= n.m^3 + \frac{n^2 + n}{2} 3m^2 a + \frac{n^2 + n.2n + 1}{2} ma^2 + \frac{n^2.n + 1^2}{4} a^3$.

VIII. The sum of n terms of a series of biquadrates, first term and common difference as above, is $n.m^4 + \frac{n^2 + n}{2} 2m^3 a + \frac{n^2 + n.2n + 1}{2} ma^2 + \frac{n^2.n + 1^2}{2} ma^3 + \frac{6n^5 + 15n^4 + 10n^3 - n}{30} a^4$.

IX. The sum of n terms of a series of rectangles $m + a$.
 $w + a + m + 2a . w + 2a + m + 3a . w + 3a + \&c$.
 $m + na . w + na$ is $= n . mw + \frac{n^2 + n}{2} m + w . a + \frac{n^2 + n.2n + 1}{6} a^2$; and if $a = 1$, will become $=$

$$\frac{n}{4} . \frac{2m + n + 1}{4} . \frac{2w + n + 1}{4} + \frac{n + 1}{3} . \frac{n - 1}{4}.$$

X. The sum of n terms of a series of parallelopipedons
 $m + a . w + a . v + a + m + 2a . w + 2a . v + 2a + \&c$.
 $m + na . w + na . v + na$ is $= n . m w v + \frac{n^2 + n}{2} m w + m v + w v . a + \frac{n^2 + n.2n + 1}{6} m + w + v . a^2 + \frac{n^2 . n + 1^2}{4} a^3$.

XI. The sum of n terms of a series of figurative numbers of the m th order is $= \frac{n}{1} . \frac{n + 1}{2} . \frac{n + 2}{3} . . . \frac{n + m - 1}{m}$.

XII. The sum of n terms of a series of polygonal numbers of the m th order is $= \frac{n^2 + n}{2} + \frac{n^3 - n}{6} m$.

XIII. The

XIII. The value of a series of reciprocals of figurative numbers of the m th order infinitely continued, is $= \frac{m-1}{m-2}$.

XIV. The sum, or value of an infinite series arising from the multiplication of the terms of a rank of figurate numbers of the m th order, into the respective terms of a geometrical progression proceeding from unity, 1, n , n^2 , n^3 , n^4 , &c. the

sum is $= \frac{1}{1-n^m}$

PROMISCUOUS EXAMPLES.

1. Required the number of cannon-shot in a triangular pyramid complete, the side of which at base, contains 36 shot?
Ans. 8436.
2. Required the number of cannon-shot in the frustum of a triangular pyramid, the side of which at top contains 12 shot, and the height 19 tiers?
Ans. 4674.
3. Required the number of shot in a square pyramid complete, the side of which at base contains 50?
Ans. 42925.
4. Required the number contained in the frustum of a square pyramid, the side of which at top contains 16 shot, and the height 25 tiers?
Ans. 20900.
5. Required the number of cannon-shot in an oblong, or triangular pile complete, the greater side at base containing 60, and the less 30 shot?
Ans. 23405.
6. Required the number of shot in an oblong frustum, consisting of 31 tiers, and the sides at top containing 24 and 16 shot respectively?
Ans. 39959.
7. Required the number of cubic feet in a square pyramid, composed of 100 cubical stones, the sides of which are in arithmetical progression, first term and common difference one foot?
Ans. 25502500.
8. Suppose 100 guineas arranged in a straight line, and let their distances from a given point in the same (taken in any measure whatever) be represented by the figurate numbers of the third order: required the distance, in the same measure which a person has to travel, who sets out from the said point to collect them thither one by one?
Ans. 33433400.
9. Suppose the distances above mentioned to have been represented

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11.

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14.

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16. Required the sums of the reciprocals

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presented by the figurate numbers of the fourth order ; required the distance to be travelled by the collector ?

Ans. 83834250500.

10. Required the number of trees in a triangular grove, each side of which measures 12 chains, and the trees planted at 6 feet asunder ?

Ans. 8911.

11. There is a grove planted with 10440 trees at 10 feet asunder ; its form is an equilateral triangle : required the side and area ? *Ans.* 144 trees in each side, and the length is = $21\frac{2}{3}$ chains, from whence the area may be found.

12. Required the number of trees in a hexagonal grove, the side of which contains 60 trees ?

Ans. 10621.

13. Required the sum of 40 terms of the hexagonal series, 1, 6, 15, 28, 45, &c. ?

Ans. 43460.

14. Required the 120th term of the octagonal series, 1, 8, 21, 40, 65, 96, &c. ?

Ans. 42960.

15. Required the sum of the duodecagons, 1, 12, 33, 64, 105, &c. to the 60th term inclusive ?

Ans. 361730.

16. Required the sums of the reciprocals

$$\left\{ \begin{array}{l} 1 + \frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \frac{1}{24} + \frac{1}{35} + \frac{1}{48}, \text{ \&c.} \\ 1 + \frac{1}{4} + \frac{1}{10} + \frac{1}{20} + \frac{1}{36} + \frac{1}{54} + \frac{1}{72}, \text{ \&c.} \\ 1 + \frac{1}{5} + \frac{1}{15} + \frac{1}{30} + \frac{1}{45} + \frac{1}{60} + \frac{1}{75}, \text{ \&c.} \\ 1 + \frac{1}{6} + \frac{1}{21} + \frac{1}{36} + \frac{1}{54} + \frac{1}{72} + \frac{1}{84}, \text{ \&c.} \\ 1 + \frac{1}{7} + \frac{1}{28} + \frac{1}{49} + \frac{1}{63} + \frac{1}{84} + \frac{1}{98}, \text{ \&c.} \\ 1 + \frac{1}{8} + \frac{1}{32} + \frac{1}{64} + \frac{1}{96} + \frac{1}{128} + \frac{1}{160}, \text{ \&c.} \\ 1 + \frac{1}{9} + \frac{1}{45} + \frac{1}{81} + \frac{1}{135} + \frac{1}{180} + \frac{1}{225}, \text{ \&c.} \\ 1 + \frac{1}{10} + \frac{1}{50} + \frac{1}{100} + \frac{1}{150} + \frac{1}{200} + \frac{1}{250}, \text{ \&c.} \\ 1 + \frac{1}{11} + \frac{1}{55} + \frac{1}{110} + \frac{1}{165} + \frac{1}{220} + \frac{1}{275}, \text{ \&c.} \\ 1 + \frac{1}{12} + \frac{1}{60} + \frac{1}{120} + \frac{1}{180} + \frac{1}{240} + \frac{1}{300}, \text{ \&c.} \\ 1 + \frac{1}{13} + \frac{1}{65} + \frac{1}{130} + \frac{1}{195} + \frac{1}{260} + \frac{1}{325}, \text{ \&c.} \\ 1 + \frac{1}{14} + \frac{1}{70} + \frac{1}{140} + \frac{1}{210} + \frac{1}{280} + \frac{1}{350}, \text{ \&c.} \end{array} \right.$$

ad infinitum

$$\left\{ \begin{array}{l} = \frac{2}{3} \\ = \frac{3}{4} \\ = \frac{4}{5} \\ = \frac{5}{6} \\ = \frac{6}{7} \\ = \frac{7}{8} \\ = \frac{8}{9} \\ = \frac{9}{10} \\ = \frac{10}{11} \\ = \frac{11}{12} \\ = \frac{12}{13} \\ = \frac{13}{14} \end{array} \right. \text{ \&c.}$$

ALTERNATION AND COMBINATION OF QUANTITIES.

ALTERNATION, OR PERMUTATION, is the mode of determining the number of changes of order in any given number of things m , taken n by n .

COMBINATION, OR ELECTION, is the mode of determining how often a less number of things n , can be taken out of a greater number m , and combined together, without regard to their order ; but with this limitation, that no two groups

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shall

shall be exactly the same. If m be $= u + v + r$, &c. the number of which is $= n$, and the several groups to be composed of one from each; then this species of combination is generally called composition.

The greater part of questions relating to the above may be solved by help of the following

THEOREMS.

I. The number of alternations of m things $a b c d$, &c. all different from each other, taken n by n , is $= \frac{m \cdot m-1 \cdot m-2 \cdot m-3 \cdot m-4 \dots m+1-n}{n}$.

II. The number of combinations of m things $a b c d$, &c. all different from each other, taken n by n , is $= \frac{m \cdot m-1 \cdot m-2 \cdot m-3 \cdot m-4 \dots m+1-n}{n \cdot n-1 \cdot n-2 \cdot n-3 \cdot n-4 \dots n+1-n}$.

III. The number of alternations of m things $au bv cr$, &c. (where u, v, r , &c. represent the number of each kind, taken n by n , is $=$ the number of alternations of the $m-1$ things $au-1 bv cr$, &c. plus the number of alternations of the $m-1$ things $au bv-1 cr$, &c. plus the number of alternations of the $m-1$ things $au bv cr-1$, &c. taken $n-1$ by $n-1$.

IV. The number of alternations of the m things au, bv, cr , &c. when $n = m$, will then become $= \frac{m \cdot m-1 \cdot m-2 \cdot m-3 \cdot m-4 \dots m+1-m}{u \cdot u-1 \cdot u-2 \cdot u-3 \cdot u+1-u \cdot v \cdot v-1 \cdot v-2 \cdot v-3 \cdot v+1-v \cdot r \cdot r-1 \cdot r-2 \cdot r-3 \cdot r+1-r}$.

V. If there be any number of things m , all different from each other, divided into n parcels containing u, v, r, s , &c. respectively; the number of compositions, each containing n things, will be $= u v r s$, &c.

EXAMPLES.

1. How many days may 12 persons be differently arranged at dinner?

Ans. 479001600.

2. How

2. How many changes may be made in the words of the following verse?

Parve (nec invideo) sine me liber ibis in urbem :

Hei mihi ! quod domino non licet ire tuo !

Ans. 355687428096000.

3. How many changes may be rung on 12 bells, taken 6 by 6?

Ans. 665280.

4. How many variations may be made of the letters in the word Bacchanalia?

Ans. 831600.

5. How many different numbers, of 10 places each, can be made of the following figures, 5443336666?

Ans. 12600.

6. How many changes can be made of the 8 letters, $a^3 b^3 c^2$?

Ans. 70.

7. How many changes can be made of these 10, $a^4 b^2 c^2 d e$, taken 8 by 8?

Ans. 22260.

8. How many combinations of 24 things, all different, taken 6 by 6?

Ans. 134596.

9. How many different files of 10 men may be made with a company of 100?

Ans. 450700595070.

10. How many combinations are there in $a^5 b^5 c^4 d^4 e^4 f^3 g^1$, taken 10 by 10?

Ans. 2819.

11. How many ways may 6 men be chosen from 6 companies, consisting of 24 men each, by taking one out of each company?

Ans. 191102976.

12. How many possible changes by throwing 12 dice?

Ans. 2176782336.

F E L L O W S H I P.

FELLOWSHIP, considered as an arithmetical operation, is the method of dividing a given quantity into any assigned number of parts, which shall be proportional to so many other proposed numbers each to each.

When there is only one set of given numbers to which the parts of the given quantity are to be proportional, it is called Single Fellowship; but when two, three, or more, i. e. when the given numbers are compounded of so many different ratios, it is then called Double Fellowship.

By Single Fellowship a bankrupt's estate may be divided among his creditors, legacies adjusted, when there is a deficiency of assets, &c.

RULE.

As the sum of the given numbers to which the required numbers must be proportional :

Is to the given quantity to be divided ::

So is each of the given numbers to which the required numbers must be proportional :

To each of the numbers required.

EXAMPLES.

1. Divide the number 3600 into 4 such parts as shall be to each other as the numbers 3, 4, 5, 6 ?

Ans. 600, 800, 1000, 1200.

2. Four merchants, A, B, C, and D, made a joint stock of 3600l, to which A paid 1200l. B 1000l. C 800l. and D 600l. ; now it so happened that they gained 900 : what was each man's share of the said gain ?—*Ans.* A's = 300, B's = 250, C's = 200, and D's = 150.

3. Five captains plundered the enemy of 12000l. The first had 20 men, the second 40, the third 55, the fourth 55, and the fifth 70. What must each captain have in proportion to his number of men ?

Ans. { 1000l. 1st.
2000l. 2d.
2750l. 3d.
2750l. 4th.
3500l. 5th.

In the greater part of questions relating to Double Fellowship, there are only two sets of proportional numbers concerned, i. e. quantity of stock, and time of continuance ; but if the several stocks be of different qualities, there arise three : they may all easily be solved by help of the following

RULE.

Multiply continually the numbers representing the quantity, quality, and time of continuance of each stock ; and the several products will be the proportional numbers to be used as in Single Fellowship.

EXAMPLES.

1. Three graziers hired a piece of land for 60l. the first put in 5 sheep for 4 months, the second put in 10 sheep for 3 months, and the third put in 8 for 5 months. What part of the rent must each pay ? *Ans.* The first 13l. 6s. the second 20l. and the third 26l. 13s. 4d.

2. A

2. A ship's company take a prize of 59400*l*. which they agree to divide among them according to their pay, and the time they have been on board: now the commissioned officers have been on board 6 months, the midshipmen 4 months, and the sailors 3 months; the commissioned officers have 44*s*. per month, the midshipmen 33*s*. and the sailors 22*s*.; moreover, there are 4 commissioned officers, 12 midshipmen, and 110 sailors. What must be the share of each officer, midshipman, and sailor? *Ans*. Each commissioned officer 264*l*. each midshipman 132*l*. and each sailor 66*l*.
3. Five graziers, A, B, C, D, and E, hired a grass field for 276*l*. A put in 6 horses 2 months, B put in 9 oxen 5 months, C put in 8 cows 10 months, D put in 45 sheep 12 months, and E put in 75 lambs 16 months: now it appeared that 2 horses devoured grass equal to 5 oxen, 3 oxen to 4 cows, 2 cows to 9 sheep, and 3 sheep to 5 lambs. What must each man pay in proportion to the grass destroyed? *Ans*. A 24*l*. B 36*l*. C 48*l*. D 72*l*. and E 96*l*.

E X C H A N G E.

EXCHANGE is the method of bartering or exchanging the money or wares of one place for that of another.

The particular customs in various nations, relating either to the exchange of money or wares, are better known to merchants and other traders, than to any in the author's situation.

The relative value of money or wares in various nations being given, any question relating to exchange may be solved by the rule of Proportion; and when the proportion is complicated, or the terms many, the following is an universal

R U L E.

Place the terms expressing the relative values of the quantities to be exchanged in two perpendicular columns, in form of so many different equations. Then multiply continually the terms on each side, and make the product of the known terms on that side where the unknown quantity is concerned a divisor, and the product of the terms on the other side a dividend: the quotient will be the value of the quantity required.

E X A M P L E S.

1. If 6*lb*. of sugar be equal in value to 7*lb*. of raisins, 5*lb*.

of raisins to 4 yards of ribband, 10 yards of ribband to 40 nutmegs, and 7 nutmegs to 18d. ; what is the value of 3lb. of sugar ? *Ans.* 16d.

2. If 3 pair of gloves be equal in value to 2 yards of lace, 3 yards of lace to 7 dozen of buttons, 6 dozen of buttons to 2 pen-knives, and 21 pen-knives to 18 pair of buckles ; how many pair of gloves is equal in value to 28 pair of buckles ? *Ans.* 63.

3. If 9 English shillings be equal in value to 2 French crowns, 1 French crown to 3 livres, 4 livres to 3 guilders, 9 guilders to 4 rix dollars, and 4 rix dollars to 3 Barcelona ducats ; what is the value of 5 Barcelona ducats in English money ? *Ans.* 30 shillings.

A COLLECTION OF PROMISCUOUS QUESTIONS.

Question 1. Three operatives, the master, the journeyman, and the apprentice, can, when working separately, perform a piece of work in different times, viz. the master in 12 days, the journeyman in 16 days, and the apprentice in 20 days. Now I demand in what time the work may be performed by their joint agency ?

Question 2. Three merchants, A, B, and C, make a joint stock of 20,000l. of which the part of A to that of B is as five to seven, and the part of B to that of C as seven to eleven ; and they find their whole gain in a certain time to be 5553l. Now I demand each man's part of the stock, and share of the gain ?

Question 3. Three merchants, A, B, and C, enter into partnership thus : A puts into the stock 781. for six months, B puts in 961. for eight months, and C puts in 1121. for twelve months ; with these they traffic, and gain 1661. 12s. It is required to find each man's share of the gain, in proportion to his stock, and time of employing it ?

Question 4. Six merchants A, B, C, D, E, and F, enter into partnership, and compose a joint stock of 36,4421. with which, in a certain time, they gain 18,000l. Now the parts advanced by A, B, C, D, E, and F, are as the numbers 1, 2, 3, 4, 5, and 6 ; and the times of their continuance as 36, 25, 16, 9, 4, and 1, respectively. I demand each man's stock, and share of the gain ?

Question 5. Two merchants, A and B, barter thus : A hath 96 yards of broad cloth, worth 10s. 6d. per yard, ready money ;

money; but in barter he values it at 12s.; B hath shalloon, worth 2s. per yard, ready money. It is required to determine the barter price of the shalloon, and number of yards to be given, so that the gain may be the same?

Question 6. A Bill of Exchange was accepted at London for the payment of 6000l. sterling, for the like value delivered in Amsterdam, at 11. 13s. 4d. per pound sterling. How much money was delivered at Amsterdam?

Question 7. When the exchange from Antwerp to London is at 11. 14s. 7d. Flemish, for 11. sterling; how many pounds sterling must be paid in London, to balance 4721. Flemish at Antwerp?

Question 8. What will 256l. 10s. amount to in 15 years, 1 quarter, 2 months, and 18 days, at 5 per centum per annum, simple interest?

Question 9. What principal being put to interest at 5 per centum per annum simple interest, will in 15 years, 6 months, and 12 days, amount to 12,365l. 10s.?

Question 10. At what rate per cent. simple interest will 875l. 12s. amount to 2626l. 16s. in 30 years?

Question 11. In what time will 4444l. amount to 8888l. at $5\frac{1}{2}$ per cent. simple interest?

Question 12. What will 256l. 10s. amount to in seven years, at 6 per cent. compound interest?

Question 13. What principal or sum of money will raise a stock of 385l. 13s. $7\frac{1}{2}$ d. in seven years at 6 per cent. compound interest?

Question 14. In what time will 256l. 10s. raise a stock of 385l. 13s. $7\frac{1}{2}$ d. at 6 per cent. compound interest?

Question 15. If 256l. 10s. raise a stock of 385l. 13s. $7\frac{1}{2}$ d. in seven years, what is the rate at compound interest?

Question 16. If 30l. yearly rent, or annuity, be forborne nine years; what will it amount to at 6 per cent. compound interest?

Question 17. What yearly rent, or annuity, being forborne nine years, will raise a stock of 344l. 14s. $9\frac{1}{2}$ d. at 6 per cent.?

Question 18. In what time will 30l. annuity raise a stock of 344l. 14s. $9\frac{1}{2}$ d. at 6 per cent.?

Question 19. If an annuity of 30l. being forborne nine years, amount to 344l. 14s. $9\frac{1}{2}$ d. what must be the rate per cent. compound interest?

Question 20. What is the present worth of 30l. per annum, to continue seven years, at 6 per cent. compound interest?

Question 21. What annuity, or yearly rent, to continue for seven years, may be purchased for 167l. 9s. 5d. allowing 6 per cent. to the purchaser?

Question 22. How long may one enjoy a lease of 30l. annuity, for 167l. 9s. 5d. allowing 6 per cent. compound interest to the purchaser?

Question 23. Suppose one should give 167l. 9s. 5d. for the purchase of a pension, or annuity, of 30l. per annum, to continue seven years; at what rate of interest per cent. would that purchase be made, allowing compound interest to the purchaser?

Question 24. What is the present worth of 75l. yearly rent, which is not to commence until ten years hence, and then to continue seven years after that time, at 6 per cent. compound interest?

Question 25. What annuity, or yearly rent, to be entered upon ten years hence, and then to continue seven years, may be purchased for 233l. 15s. 9d. ready money, at 6 per cent. compound interest?

Question 26. Suppose an annuity or yearly rent of 75l. to commence ten years hence, and from that time to continue seven years, may be purchased for 233l. 15s. 9d. ready money; at what rate of interest is the purchase made, granting compound interest to the purchaser?

Question 27. My friend hath drawn upon me for 3000l. to be paid at different times, viz. 300l. at three months, 600l. at six months, 900l. at nine months, and 1200l. at twelve months. I demand at what time I may, with justice to myself and the holder of the bills, discharge the same at one payment?

Question 28. An hundred guineas being placed in a straight line, the first distant from the second 3 yards, the second from the third 5 yards, the third from the fourth 7 yards, &c. and suppose a box to be placed in the same line produced, one yard distant from the first: how far must a man travel to collect them one by one into the box?

Question 29. Suppose the distances of the guineas from the box to be 1, 8, 27, 64, &c. yards respectively; and that they are collected one by one as before: how far must the collector travel, and in what time will he collect them, travelling at the rate of four miles per hour?

Question 30. A certain nobleman having a suit of superfine clothes, with 12 dozen of silver-plate buttons, and being

being asked their price, replied, that if one farthing was laid upon the first button, two upon the second, four upon the third, and so on; he would not only give the clothes, but also his whole estate. I demand the value of the clothes and estate?

Question 31. A certain gentleman had two horses worth 100*l.* and a saddle worth 50*l.* Now the horses being of different values, if the saddle be put upon the worst horse, it will make its value in proportion to that of the best, as 3 : 2; but if put upon the best horse, it will make its value in proportion to that of the worst, as 11 : 4. What was the value of each?

Question 32. A certain poor school-master had so many scholars, that if every one would have contributed 5*l.* there would only want 30*l.* to buy him a good house; but if each scholar would have contributed 6*l.* there would be 40*l.* more than enough. I demand the number of scholars, and price of the house?

Question 33. I am a brazen lion: my two eyes, my mouth, and the sole of my right foot are so many several pipes, which flow uniformly, and fill a cistern in different times, viz. my right eye can fill it in two days, my left eye in three, and the sole of my right foot in four; but my mouth can fill it in six hours. Pray tell me in what time all these together, my mouth, my eyes, and my foot, will fill this cistern?

Question 34. What fraction is that whose square exceeds its cube by the greatest quantity possible?

Question 35. What number is that whose half, third, and fourth part make 26?

Question 36. What number is that whose third part exceeds its fourth part by 12?

Question 37. There are two numbers the least in the same proportion, and the square of the greater exceeds the cube of the lesser by 22; what are these numbers?

Question 38. There is a wall whose length is double its height, and its height quintuple its breadth; and it contains 1350 cubic feet. I demand the length, height, and breadth of this wall?

Question 39. We read that formerly between the mountain Thornax and the town Halix, there was a trench dug, the length of which was thirty times its breadth, and the breadth triple its depth; its whole capacity was supposed to be 13840 cubic feet: what were the particular dimensions of the trench?

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Question 40. A certain gentleman gives to the first beggar he meets $\frac{1}{6}$ th part of the pence that he had about him, and also 4d. more; to the second $\frac{1}{6}$ th of the remaining pence, and 8d. more; and after the same manner continuing to give $\frac{1}{6}$ th of the remainder, with an additional sum increasing every time by 4d. until he had given away all his pence, and then found that every one had received equal alms. The question is to know how many pence he had, and also the number of poor people.

Question 41. Cupid complained to his mother, that the Muses had taken away his apples; Clio, saith he, hath taken away $\frac{1}{5}$ th; Euterpe $\frac{1}{12}$ th; Thalia $\frac{1}{8}$ th; Melpomene $\frac{1}{20}$ th; Erato $\frac{1}{7}$ th; Terpsichore $\frac{1}{4}$ th; Polymnia $\frac{3}{10}$; Urania $\frac{1}{5}$; and Calliope, the most spiteful of them all, hath taken $\frac{3}{10}$; so that I have but five apples left. Query, how many apples had he at first?

Question 42. At Thebes, in the street which was called Prætidis, stood the temple of Euclea, seventy-six feet high; opposite to which stood the temple of Boedromius Apollo, which was fifty-seven feet high; and the distance between them both was supposed to be one hundred and fourteen feet. Between these two temples was placed the statue of a lion cut in marble, equidistant from the top of each temple, which was said to be dedicated by Hercules, when he had defeated Erchinus, king of the Orcaomenians: what was the distance of the lion from the base of each temple?

Question 43. The Spartans hired these three famous painters, Arcefilaus, Euphranius, and Onasia, to adorn with pictures the Lesche Crotanorum, or senate-house of the Piræaters. They agreed to give Arcefilaus forty drachms per diem; Euphranius was to have fifty, and Onasia sixty: now these three artificers finished the work in 120 days, and when they were to be paid for their work according to their bargain, all three received equal sums. The question is, to know how many days, according to this proportion, each man spent in finishing his work.

Question 44. The temple of the three Graces at Athens stood upon a rectangular area, the length of which exceeded its breadth by three paces; and the distance from one angle to that which was diametrically opposite to it, exceeded the length also by so many paces. How much was its length, breadth, and diametrical distance?

Question 45. Buporthmos was a promontory of Blopon-
nesus,

nefus, running out into the sea, not far from the island Tricana, opposite to the gulf of Scylla. On this promontory stood a temple 40 paces long, divided into two unequal areas: the greater was dedicated to Ceres, the lesser to Proserpine. The capacity of Ceres's was 192 square paces; Proserpine's was twice as long as it was broad: what was the breadth of this temple?

Question 46. In the Danish annals, we have an incredible and unusual instance of the sincere love that was between Hagbarus and Signa, the children of two neighbouring princes, who were at war with one another. When Signa heard that her incensed father had crucified her lover Hagbarus, she, together with her virgin companions, first set on fire the king's palace, and then, fastening their girdles to one of the beams, hanged themselves. The ruins of this royal palace are to be seen at this day in Seeland, not far from Signaria, formerly a stately castle, but now a small village of no great note. The surface of this place contains 5400 square feet, and its length exceeds its breadth by 250 feet: what is its length and its breadth?

Question 47. In the city of Megara, in the way through Jupiter's Wood to the Castle Caria, were to be seen two little square temples, the one dedicated to Nyctelius Bacchus, and the other to Apostrophia Venus: their pavements were laid with stones a foot square each; but the side of Venus's temple exceeded that of Bacchus's by 12 feet, and both their pavements taken together contained 2120 stones. What was the length of them separately?

Question 48. Two troops or parties of soldiers have each of them an equal number of crowns to be distributed amongst them. In one troop there are four men more than in the other. Now, the money being divided, each man in the lesser troop hath eight crowns more than those in the greater; and the number of crowns to be distributed amongst both troops exceed the number of men by 172. I demand the number of men, and crowns distributed.

Question 49. Two captains distribute each of them 1200 crowns between a certain number of soldiers. One has 40 soldiers less than the other; and it is found that those of the lesser number received every one five crowns a-piece more than those of the greater number. The question is, to know how many soldiers each captain had.

Question 50. In the shire of Tiviotdale, near Jedburgh, stand

stand the ruins of Cesford Castle, the ancient seat of the Kers, now Dukes of Roxburgh. Near the foot of this castle wall there grew a beautiful tall pine, which being cut down by a certain person, the top struck the castle wall 36 feet from the ground. Now the distance of the pine's root from the bottom of the wall, was to the distance of its top from the same point, as 4 : 3, and to the whole height of the wall as 3 : 5. I demand the length of the pine, and height of the wall.

Question 51. Walking one evening on the sandy shore,
Where shell fish breed, and seas for ever roar,
I saw a ship dance on the rolling tide,
With curling smoke advancing from her side ;
Which for some moments my dull sense employs,
Before I heard the thundering cannon's noise :
Then from my job time's register I drew,
And from her sides a second lightning flew ;
Full fourteen seconds of swift time expir'd
Before my list'ning ears the sound admir'd.
Ye learned heads who sound's progression know,
My distance from the floating vessel shew ?

Question 52. When the mercury stands at the height of 30 inches in the common barometer, what will be the height of the mercury in a tube of 3 feet, with 6 inches of common air in the top ?

Question 53. A hare being 50 paces before a grey-hound, makes four leaps to the dog's three ; but two leaps of the dog are as much as three of the hare. How many leaps must the grey-hound take to catch the hare ?

Question 54. In three bags there is contained a certain number of guineas. The sum of the guineas in the first and second bags is 20 ; the sum of the guineas in the second and third bags is 48 ; and the sum of the guineas in the first and third bags is 44. What number of pounds was in each ?

Question 55. There was a wood, or grove, in the vicinity of Phares, a city of Achaia ; which grove, it is said, Telandes measured, and found its length to be double its breadth ; also the number of superficial paces in the area, added to the number of lineal paces in the length and breadth, to be 1430.—What was the length and breadth of the grove ?

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